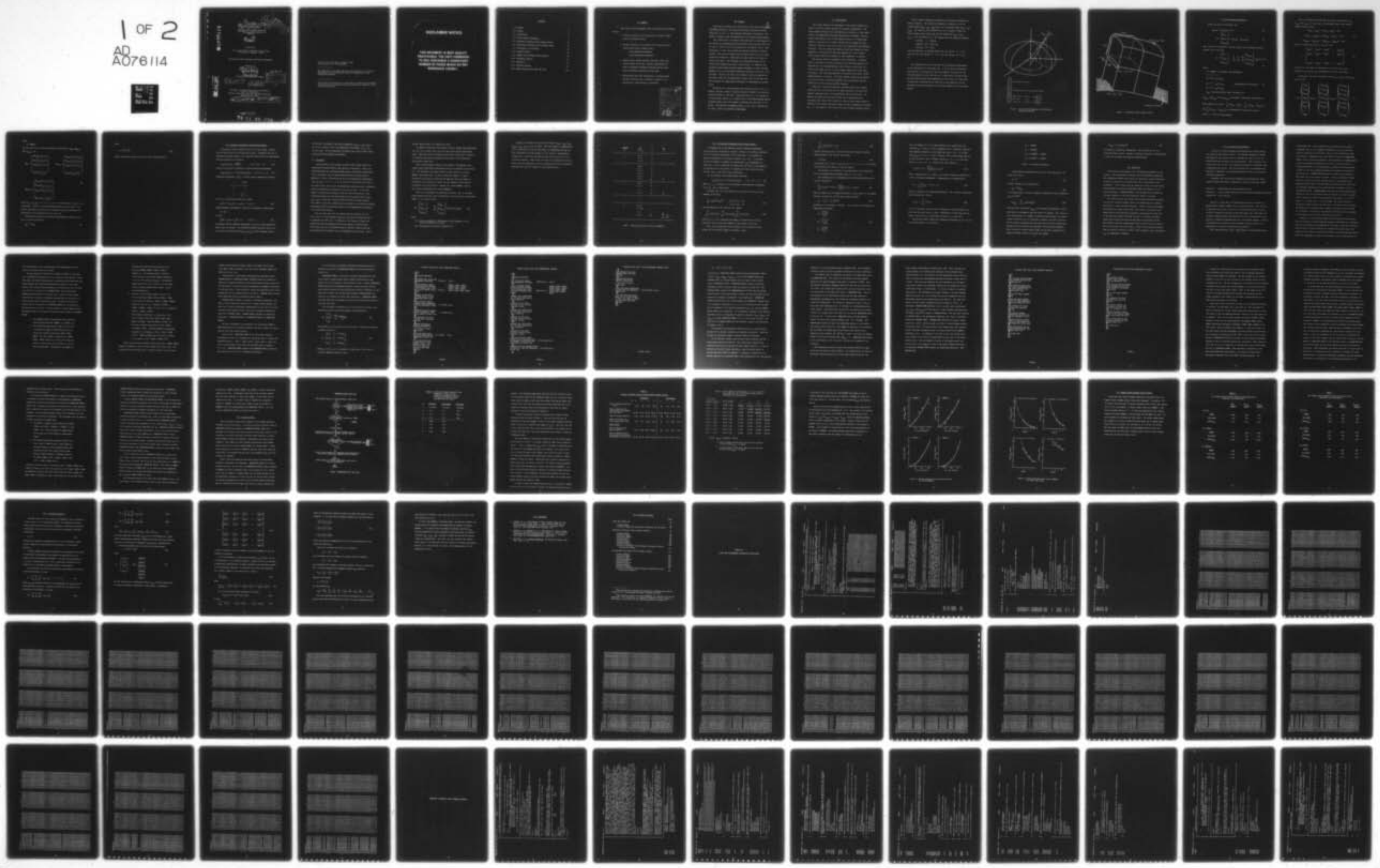
AD-A076 114 VIRGINIA POLYTECHNIC INST AND STATE UNIV BLACKSBURG --ETC F/G 8/5
FINITE ELEMENT MODELS OF THE EARTH'S GRAVITY FIELD.(U)
OCT 79 R C ENGELS , J L JUNKINS DAAK70-78-C-0072

UNCLASSIFIED ETL-0198 NL

1 OF 2
AD
A076114



AD A076114

LEVEL II

(12)

(9) Final Report. on Phase 4

(6) FINITE ELEMENT MODELS

OF THE EARTH'S GRAVITY FIELD

PHASE IV

(18) ETL (19) 0198

Prepared for

U.S. Army Engineer Topographic Laboratories
Fort Belvoir, Virginia 22060

Approved for Public Release; Distribution Unlimited

Submitted by:

(10) Remi C. Engels
Assistant Professor

John L. Junkins
Professor, Principal Investigator

(11) 5 Oct 79

(12) 152

Department of Engineering Science and Mechanics
College of Engineering
Virginia Polytechnic Institute and State University
Blacksburg, VA 24061

(15) DAAK70-78-C-0072

404 722

79 11 05 028

Destroy this report when no longer needed.
Do not return it to the originator.

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

The citation in this report of trade names of commercially available products does not constitute official endorsement or approval of the use of such products.

DISCLAIMER NOTICE

CONTENTS

1.0 Summary

This research and development effort has produced the following results:

* Software suitable for the construction of finite element gravity disturbance fields.

* Software suitable for calculation of the gravity disturbances within finite element fields:
    - using Chebyshev polynomials
    - using Orthonormal polynomials

* Example finite element gravity disturbance fields have been developed and tested. Relevant comparisons between the Chebyshev polynomials approach and the Orthonormal polynomials approach have been made.

* Optimizations have been incorporated to increase computational efficiency and to determine a model for the geopotential, using Orthogonal polynomials.

2.0 Preface

This report constitutes the final report under Contract No. DAAK70-78-C-0072 performed by the Virginia Polytechnic Institute and State University for the U. S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia, under the sponsorship of the Defense Mapping Agency.

The current effort is primarily concerned with the investigation of new choices of computationally more efficient basis functions which can be used to model the local fine structure of the gravity vector. This development, to date, has produced two separate versions of software systems. This report documents each of these systems as well as their theoretical background. Comparisons are made based on the result of several test cases. Also included are detailed discussions of the software and standard print-outs of each version. In addition, the programs contain enough detailed comment statements to allow the knowledgeable user to use and modify particular sections to suit his or her own needs. Finally, the report also indicates a possible approach to determine a model for the geopotential itself, which at the same time would increase the computational efficiency of determining the gravity vector.

The authors very much appreciate the technical liaison of Mr. L. A. Gambino, Director, Computer Sciences Laboratory (USAETL), who served as technical monitor of this work. Discussions with Dr. R. W. Ballew, Dr. B. Louis Decker, and Mr. H. W. Howard (all of the Defense Mapping Agency, Aerospace Center) were also helpful in defining the objectives of our effort. The excellent programming support of Mr. John T. Saunders and Mr. John J. Smith is gratefully acknowledged.

3.0 Introduction

The central theme in the development of the present software systems is the finite element approach to piecewise approximation. The finite element concept may be considered as an extension to three dimensions of one dimensional piecewise approximation techniques. Qualitatively, some quantity that is functionally related in a "complicated" fashion to position over a "large" interval may be replaced for much "smaller" intervals of position by much "less complicated" approximating functions. Thus the complicated function, which will normally be "more expensive" to evaluate, will have been replaced by a set of locally valid functions which will normally be "less expensive" to evaluate.

The idea may easily be extended to three dimensions. A quantity which is functionally related to position over some large region may be replaced by a set of approximations, each of which is valid only in a small local volume; the set of local volumes spanning the large region of interest. The local approximations are typically much easier (faster computationally) to evaluate. The finite element concept may be applied to any of a variety of modeling problems; this investigation centered specifically on fine structure gravity modeling.

Previously, earth-fixed spherical coordinate based finite element fields have been successfully used to replace globally valid spherical harmonic representations of the geopotential and its derivatives. But because the model (in the present study) used to simulate gravity disturbance data (Model 310) consists of a set of point masses placed in relation to the Geodetic Reference Surface of 1967, ellipsoidal coordinates (H, λ, ϕ - see Figure 1) were used in the finite element modeling process.

Finite element fields may be defined by ellipsoidal coordinates as shown in Figure 2. The region to be modeled is defined by a set of maximum bounds (H_{max}, λ_{max}, ϕ_{max}) and a set of minimum bounds (H_{min}, λ_{min}, ϕ_{min}). The region is then divided into a set of smaller volumes, or finite element cells, where each cell has dimensions H_{cell}, λ_{cell}, ϕ_{cell}. Thus the 300 km × 10^{o} × 10^{o} region above the reference ellipsoid,

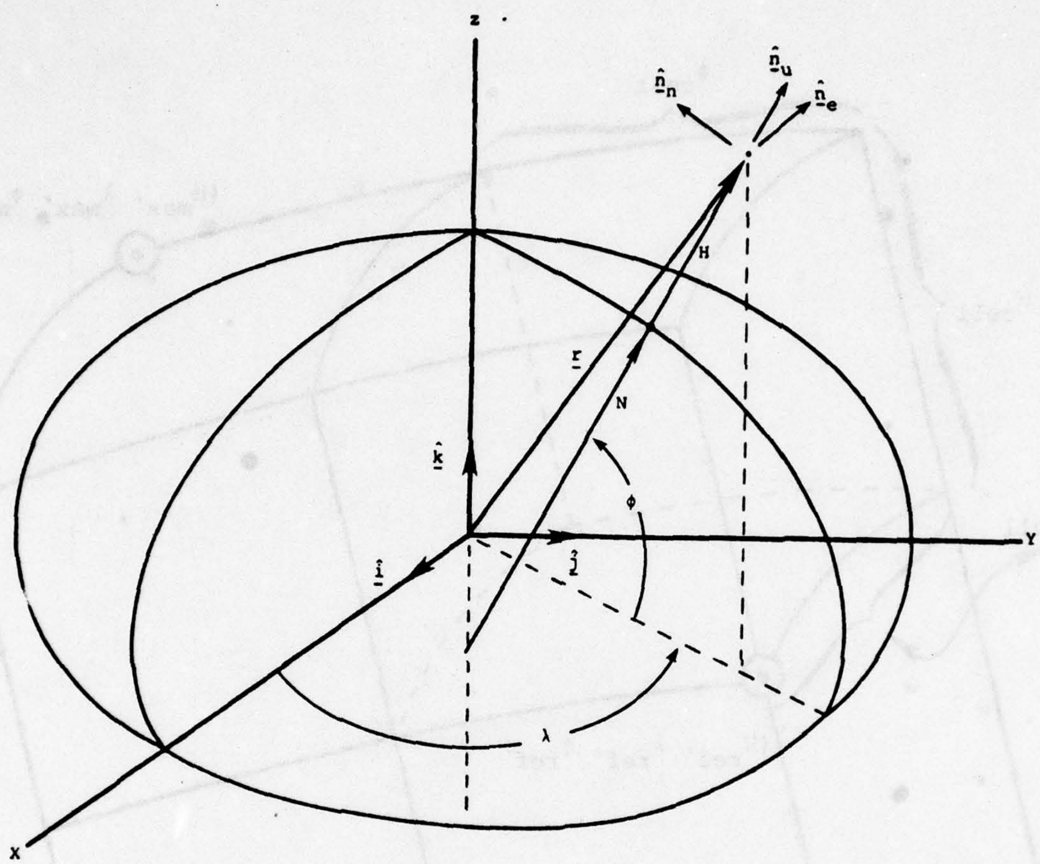
altitude 0 - 300 km,

longitude 70^{o}E - 80^{o}E, and

latitude 25^{o}S - 35^{o}S,

could be modeled by one hundred (100) cells each 300 km × 1^{o} × 1^{o} or four (4) cells each 300 km × 5^{o} × 5^{o} or one cell 300 km × 10^{o} × 10^{o}, etc.

The approximation valid in any one cell is generated independently of the approximation for any other cell. A set of gravity disturbance "observations" are produced for each cell directly from Model 310 and then approximated for only that cell in a least squares fitting process. The approximating models thus generated are then stored on a high speed rotating mass storage device for later use in the computation of the approximate gravity disturbance in any finite element cell of the region modeled.



$$\left\{\begin{matrix} \hat{\underline{i}} \\ \hat{\underline{j}} \\ \hat{\underline{k}} \end{matrix}\right\} = \text{earth-fixed unit vectors}$$

$$\left\{\begin{matrix} \hat{\underline{n}}_u \\ \hat{\underline{n}}_e \\ \hat{\underline{n}}_n \end{matrix}\right\} = \text{instantaneous up, east, and north unit vectors.}$$

$$\left\{\begin{matrix} \hat{\underline{n}}_u \\ \hat{\underline{n}}_e \\ \hat{\underline{n}}_n \end{matrix}\right\} = \begin{bmatrix} \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \\ -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \end{bmatrix} \left\{\begin{matrix} \hat{\underline{i}} \\ \hat{\underline{j}} \\ \hat{\underline{k}} \end{matrix}\right\}$$

Figure 1 Earth-Fixed Rectangular and Ellipsoidal Coordinate Systems.

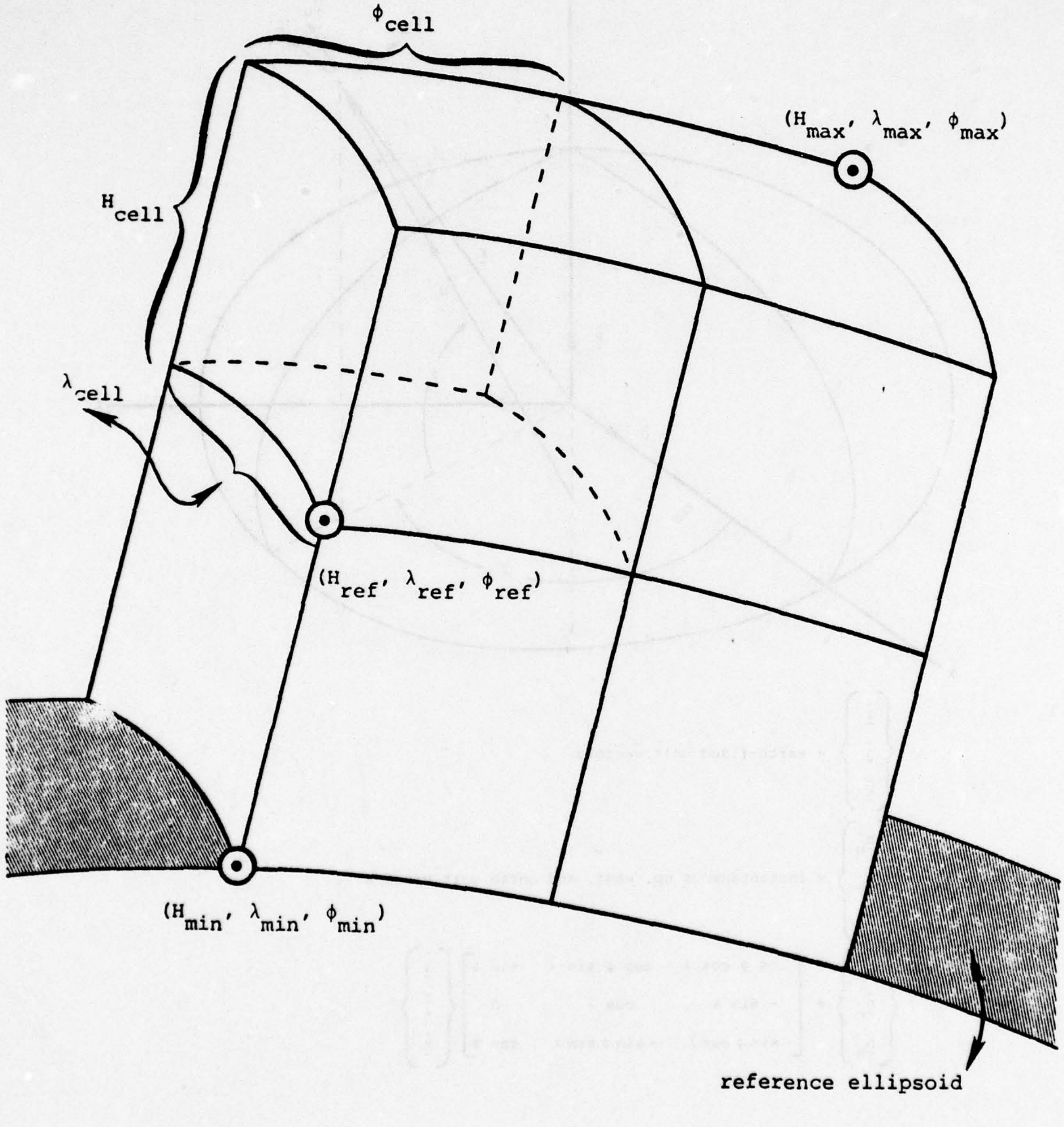


Figure 2 Ellipsoidal Finite Element Field.

4.0 Finite Element Formulation

Define the gravity disturbance, $\delta \underline{g}$:

$$\underline{g}_{actual} = \underline{g}_{reference} + \delta \underline{g} \tag{1}$$

$$\delta \underline{g} = \begin{Bmatrix} \delta g_{up} \\ \delta g_{east} \\ \delta g_{north} \end{Bmatrix} = \underline{g}_{actual} - \underline{g}_{reference} \tag{2}$$

Now, replace the components of $\delta \underline{g}$ with locally valid polynomial approximations, $\delta \underline{G}$, of the form,

$$\delta \underline{G} = \begin{Bmatrix} \delta G_{up} \\ \delta G_{east} \\ \delta G_{north} \end{Bmatrix} = \sum_{n=0}^{N} \sum_{i=0}^{n} \sum_{j=0}^{n-i} \begin{Bmatrix} C_{up_{ijk}} \\ C_{east_{ijk}} \\ C_{north_{ijk}} \end{Bmatrix} f_{ijk}(X_1, X_2, X_3) \tag{3}$$

where,

N = NORDER = the highest order polynomial,

$k = n - i - j$

$X_1 = (H - H_{ref})/H_{cell}$

$X_2 = (\lambda - \lambda_{ref})/\lambda_{cell}$ (non-dimensional coordinates) (4)

$X_3 = (\phi - \phi_{ref})/\phi_{cell}$

f_{ijk} = chosen polynomial basic functions, and

$C_{up_{ijk}}$, $C_{east_{ijk}}$, and $C_{north_{ijk}}$ are constant coefficients determined via least square fits so that $\sum_{i=1}^{m} (\delta G_{up} - \delta g_{up})_i^2$, $\sum_{i=1}^{m} (\delta G_{east} - \delta g_{east})_i^2$, and $\sum_{i=1}^{m} (\delta G_{north} - \delta g_{north})_i^2$ are minimized for some local volume of space (i.e. the cell being modeled).

Thus, to determine the three sets of constant coefficients, C_{up}, C_{east}, and C_{north}, for each cell, the following linear least squares problems must be solved,

$$||\delta \underline{G}_{up} - \delta \underline{g}_{up}|| = ||A\underline{C}_{up} - \delta \underline{g}_{up}|| = \text{min.}$$

$$||\delta \underline{G}_{east} - \delta \underline{g}_{east}|| = ||A\underline{C}_{east} - \delta \underline{g}_{east}|| = \text{min.} \tag{5}$$

$$||\delta \underline{G}_{north} - \delta \underline{g}_{north}|| = ||A\underline{C}_{north} - \delta \underline{g}_{north}|| = \text{min.}$$

where the coefficient matrix, A, has the form,

$$A = \begin{bmatrix} f_{000}(1) & f_{001}(1) & f_{010}(1) & \dots & f_{N00}(1) \\ f_{000}(2) & f_{001}(2) & f_{010}(2) & \dots & f_{N00}(1) \\ & & \dots & & \\ f_{000}(m) & f_{001}(m) & f_{010}(m) & \dots & f_{N00}(m) \end{bmatrix} \tag{6}$$

in which m is the number of observations in the cell and $f_{ijk}(\ell)$ represents $f_{ijk}(X_{1_\ell}, X_{2_\ell}, X_{3_\ell})$ corresponding to the ℓth observation.

Furthermore, the three coefficient vectors, $\underline{C}_{up}$, $\underline{C}_{east}$, and $\underline{C}_{north}$, have the form

$$\underline{C}_{up} = \begin{Bmatrix} C_{up_{000}} \\ C_{up_{001}} \\ C_{up_{010}} \\ C_{up_{100}} \\ C_{up_{002}} \\ C_{up_{011}} \\ \vdots \\ C_{up_{N00}} \end{Bmatrix}, \underline{C}_{east} = \begin{Bmatrix} C_{east_{000}} \\ C_{east_{001}} \\ C_{east_{010}} \\ C_{east_{100}} \\ C_{east_{002}} \\ C_{east_{011}} \\ \vdots \\ C_{east_{N00}} \end{Bmatrix}, \underline{C}_{north} = \begin{Bmatrix} C_{north_{000}} \\ C_{north_{001}} \\ C_{north_{010}} \\ C_{north_{100}} \\ C_{north_{002}} \\ C_{north_{011}} \\ \vdots \\ C_{north_{N00}} \end{Bmatrix}. \tag{7}$$

where

$$N = \text{NORDER}$$

and the three sets of gravity disturbance observations, $\delta \underline{g}_{up}$, $\delta \underline{g}_{east}$, and $\delta \underline{g}_{north}$, are

$$\delta \underline{g}_{up} = \begin{Bmatrix} \delta g_{up}(X_{1_1}, X_{2_1}, X_{3_1}) \\ \delta g_{up}(X_{1_2}, X_{2_2}, X_{3_2}) \\ \delta g_{up}(X_{1_3}, X_{2_3}, X_{3_3}) \\ \vdots \\ \delta g_{up}(X_{1_m}, X_{2_m}, X_{3_m}) \end{Bmatrix} \qquad \delta \underline{g}_{east} = \begin{Bmatrix} \delta g_{east}(X_{1_1}, X_{2_1}, X_{3_1}) \\ \delta g_{east}(X_{1_2}, X_{2_2}, X_{3_2}) \\ \delta g_{east}(X_{1_3}, X_{2_3}, X_{1_3}) \\ \vdots \\ \delta g_{east}(X_{1_m}, X_{2_m}, X_{3_m}) \end{Bmatrix}$$

$$\delta \underline{g}_{north} = \begin{Bmatrix} \delta g_{north}(X_{1_1}, X_{2_1}, X_{3_1}) \\ \delta g_{north}(X_{1_2}, X_{2_2}, X_{3_2}) \\ \delta g_{north}(X_{1_3}, X_{2_3}, X_{3_3}) \\ \vdots \\ \delta g_{north}(X_{1_m}, X_{2_m}, X_{3_m}) \end{Bmatrix} \tag{8}$$

where $\{(X_{1_i}, X_{2_i}, X_{3_i}),\ i = 1, 2, \ldots, m\}$ = a specified set of (H_i, λ_i, ϕ_i) coordinates - usually a uniform observation grid in any one cell. The gravity disturbance observations are determined directly from Model 310 evaluations on the observation grid in each cell.

Following the standard least squares approximation procedure we obtain, for the "up-direction"

$$\underline{C}_{up} = B\ \delta \underline{g}_{up} \tag{9}$$

where

$$B = (A^T\ A)^{-1} A^T \tag{10}$$

Similar results are valid in the "east- and north-directions".

5.0 Chebyshev Polynomials Finite Element Models

The choice of basis functions in Eq. (3) is still open. Several sets of basis functions have been used to date. Polynomial sets are of particular interest because of the ease with which they can be manipulated. The most obvious set is given by

$$f_{ijk}(X_1, X_2, X_3) = X_1^i X_2^j X_3^k \qquad i,j,k = 0,1,\ldots,N \qquad (11)$$

Another set consists of products of shifted Chebyshev polynomials

$$f_{ijk}(X_1, X_2, X_3,) = T_i(X_1) T_j(X_2) T_k(X_3) \qquad i,j,k = 0,1,\ldots,N \qquad (12)$$

Chebyshev polynomials, $t_n(X_1)$, of order n may be generated as follows:

n	$t_n(X)$
0	1
1	X
2	$2X^2 - 1$

For $n \geq 2$, a recursion formula may be used:

$$t_n(X) = 2X\, t_{n-1}(X) - t_{n-2}(X), \quad -1 \leq X \leq 1 \qquad (13)$$

Shifted Chebyshev polynomials, $T_n(\bar{X})$ may be computed by substituting

$$X = 2\bar{X} - 1$$

so that,

$$T_n(\bar{X}) = t_n(X) = t_n(2\bar{X} - 1), \qquad 0 \leq \bar{X} \leq 1 \qquad (14)$$

To generate shifted Chebyshev polynomials a set of X's with values between 0 and 1 are needed. The difference between some point (H, λ, ϕ) in a cell and a reference point $(H_{ref}, \lambda_{ref}, \phi_{ref})$ (the "lowermost corner"

of the cell) is divided by the cell's dimensions, H_{cell}, λ_{cell}, ϕ_{cell}, as in (4) to obtain a set of non-dimensional coordinates, X_1, X_2, X_3, for each cell. The non-dimensional coordinates may then be used to generate the shifted Chebyshev polynomials.

5.1 Discussion

Certain aspects of the process by which finite element fields are created may be capitalized on under special circumstances. When it is known beforehand that uniformly gridded gravity disturbance observations will be available, only one least squares matrix (A matrix) need be generated. This is true because the set of non-dimensional coordinates, $\{(X_{1_i}, X_{2_i}, X_{3_i}),\ i = 1, 2, ..., m\}$ of one cell's observation grid will be the same as every other cell's non-dimensional observation grid coordinates when the observation grid pattern is set for all cells. If the same number of observations are made in each cell and the positions of the observations points are the same relative to the reference point (H_{ref}, λ_{ref}, ϕ_{ref}) in each cell, then all cells will have the same A matrix. The A matrix, after all, consists only of products of the locally evaluated polynomials, which are functions only of the non-dimensional coordinates of the observation points.

This all means that only one A-matrix need be generated for the first cell of a finite element field; but the gravity observations must be generated for each cell. A reduction of the A-matrix to upper triangular form may be performed for the least squares fitting process, after which each set of observations may be similarly reduced and then back-substituted to produce a set of coefficients for each cell. The A-

matrix, again, needs to be reduced only once.

It should be noted that this method of finite element field generation does not rely on Model 310 or any other particular gravity disturbance model. Model 310 could be replaced by any other process that is capable of producing the gravity disturbance observations at the observation grid points of each cell.

Once a finite element field has been created, the Chebyshev polynomial gravity disturbance approximation $\delta \underline{G}$ may be computed by evaluating (3). The subscripts ijk always follow the same pattern for a given NORDER. This means that a "current coefficient number", ℓ, may be attached to each group of subscripts ijk <u>and</u> that the total number of coefficients, NC, necessary to model any one component of the gravity disturbance in one cell will be constant for a given NORDER. Table 1 shows these relationships for various NORDER's.

Since we can precompute and save the subscripts, ijk, in the appropriate pattern such that they are functions of only the current coefficient number, ℓ, (3) can be written,

$$\delta \underline{G} = \begin{Bmatrix} \delta G_{up} \\ \delta G_{east} \\ \delta G_{north} \end{Bmatrix} = \sum_{\ell=1}^{NC} \begin{Bmatrix} C_{up_\ell} \\ C_{east_\ell} \\ C_{north_\ell} \end{Bmatrix} T_i(X_1) T_j(X_2) T_k(X_3) \qquad (15)$$

where,

NC = the total number of coefficients for one component of the gravity disturbance, as above

ijk = precomputed and saved as functions of ℓ

X_1, X_2, X_3 = functions of H, λ, ϕ and the constants H_{ref}, λ_{ref}, ϕ_{ref}, H_{cell}, λ_{cell}, ϕ_{cell} as in (4) above. The total number of summations in (15) is the same as in (3) but the amount of bookkeeping has been cut down tremendously by doing business in this fashion. Computational savings of up to 50% were obtained by using (15) in lieu of (3) for evaluation purposes. Table 1 was also used in the generation of the A-matrix, but the saving here is not as significant due to the previously discussed fact that the A-matrix is only generated once.

NORDER	ijk	ℓ	NC
0	0 0 0	1	1
1	0 0 1	2	
	0 1 0	3	
	1 0 0	4	4
2	0 0 2	5	
	0 1 1	6	
	0 2 0	7	
	1 0 1	8	
	1 1 0	9	
	2 0 0	10	10
3	0 0 3	11	
	0 1 2	12	
	⋮ ⋮ ⋮	⋮	
	3 0 0	20	20
⋮	⋮ ⋮ ⋮	⋮	⋮
N	0 0 N	.	
	0 1 N-1	.	
	⋮ ⋮ ⋮	⋮	
	N 0 0	NC	$\sum_{n=1}^{N+1} \sum_{m=1}^{n} m$

Table 1 Subscript Patterns for Various NORDER's.

6.0. Orthonormal Polynomials Finite Element Models

As demonstrated by the numerical results, Chebyshev polynomials yield an entirely satisfactory solution. However, by choosing a judicious set of orthonormal polynomials as basis functions, it is possible to avoid the computation of the matrix inverse in Eq. (10). In addition, it will be possible to compute the coefficients c_{ijk} independently of each other. This is especially convenient when the order of approximation is increased for no need exists to recompute the least squares operator B in Eq. (10), or any lower order coefficients.

Let us introduce the following basis functions

$$f_{ijk}(X_1,X_2,X_3) = f_i(X_1)f_j(X_2)f_k(X_3) \qquad i,j,k = 0,1,\ldots,N \tag{16}$$

Where f_i, f_j, f_k represent (to be specified) one-dimensional polynomials in X_1, X_2, and X_3 respectively.

Using Eq. (6), it is not difficult to show that the matrix A^TA has elements of the form

$$\sum_{\ell=1}^{m} f_{ijk}(\ell)\, f_{\alpha\beta\gamma}(\ell) \quad ; \quad \begin{array}{l} i,j,k = 0,1,\ldots,N \\ \alpha,\beta,\gamma = 0,1,\ldots,N \end{array} \tag{17}$$

and introducing Eq. (16) into Eq. (17) yields

$$\sum_{\ell_1=1}^{m_1} f_i(\ell_1)f_\alpha(\ell_1) \sum_{\ell_2=1}^{m_2} f_j(\ell_2)f_\beta(\ell_2) \sum_{\ell_3=1}^{m_3} f_k(\ell_3)f_\gamma(\ell_3) \tag{18}$$

where m_i (i = 1,2,3) represents the number of observations in the X_i-direction, and m is the total number of observations in the cell.

Next, let us assume that the set $f_i(X_1)$ can be constructed orthonormal over the discrete domain of interest. Hence,

$$\sum_{\ell_1=1}^{m_1} f_i(\ell_1) f_\alpha(\ell_1) = \delta_{i\alpha} \tag{19}$$

Similar expressions can be written for the sets $f_j(X_2)$ and $f_k(X_3)$. Substituting Eq. (19) into Eq. (18) yields

$$\delta_{i\alpha}\delta_{j\beta}\delta_{k\gamma} \tag{20}$$

and because (20) is unity if and only if $i = \alpha$, $j = \beta$, $k = \gamma$, we conclude that the matrix $A^T A$ reduces to the unit matrix.

The problem is now reduced to the construction of an orthonormal set of one-dimensional polynomials $f_i(X)$ $(i = 0,1,\ldots,N)$.

Any orthogonal set of polynomials has the following three-term recurrence relation[3]

$$\frac{h_i}{h_{i+1}} f_{i+1}(x) = x f_i(x) - \frac{g_i h_{i-1}}{g_{i-1} h_i} f_{i-1}(x) - B_i f_i(x) \tag{21}$$

where h_i stands for the leading coefficient in $f_i(x)$ and B_i is an unknown constant, and where we use the inner product notation

$$g_i \equiv \langle f_i, f_i \rangle = \int_D f_i^2(x)\,dx \tag{22}$$

and where D is the domain of interest. If we choose the following non-dimensional coordinates

$$X_1 = \frac{2(H-H_m)}{H_{cell}}$$

$$X_2 = \frac{2(\lambda-\lambda_m)}{\lambda_{cell}} \tag{23}$$

$$X_3 = \frac{2(\phi-\phi_m)}{\phi_{cell}}$$

then, the domain D is (-1,1) which enables us to conclude that the functions $f_0 = 1$ and $f_1 = x$ are orthogonal over D. Moreover, using the Gram-Schmidt procedure it can be shown that $f_{2k}(k = 0,1,...)$ is even while $f_{2k+1}(k = 0,1,...)$ is odd. From Eq. (21) it then follows that all B_i must be equal to zero. Finally, because $h_0 = h_1 = 1$, we also must have $h_i = 1$ $(i = 2,3,...)$. Relation (21) then reduces to

$$f_{i+1}(x) = xf_i - \frac{\langle f_i,f_i\rangle}{\langle f_{i-1},f_{i-1}\rangle} f_{i-1}(x) \tag{24}$$

which, together with $f_0 = 1$ and $f_1 = x$, yields a recurrence relation for a set of orthogonal polynomials. Normalization is accomplished by

$$\hat{f}_i = \frac{f_i}{\langle f_i,f_i\rangle^{1/2}} \quad i = 0,1,...,N \tag{25}$$

where $\hat{f}_i$ represents the normalized polynomial. For a discrete observation grid we replace Eq. (22) by

$$\langle f_i,f_i\rangle = \sum_{\ell_1=1}^{m_1} f_i^{\,2}(\ell_1) \tag{26}$$

It is also necessary that the local coordinates of the grid points in a particular direction add up to zero. Furthermore, we must have that the number of grid points in a particular direction is at least N+1.

Specifically Table 2 gives the orthonormal polynomials $f_i(i = 0,1,2,3)$ for the grid points (-1,-1/3,1/3,1).

$$\hat{f}_0 = .500000$$

$$\hat{f}_1 = .670820X$$

$$\hat{f}_2 = 1.125000X^2 - .625000$$

$$\hat{f}_3 = 2.51558X^3 - 2.29197X$$

Table 2 Orthonormal Polynomials

Introducing the polynomials (25) into Eq. (16) and into Eq. (6) reduces Eq. (10) to

$$B = A^T \tag{27}$$

so that, from Eq. (9) it follows that

$$\underline{C}_{up} = A^T \delta \underline{g}_{up} \tag{28}$$

It is then possible to write an explicit expression for each coefficient c_{ijk} as follows

$$c_{ijk_{up}} = \sum_{\ell=1}^{m} f_{ijk}(\ell)\delta g_{up}(\ell) \tag{29}$$

Note that each coefficient $c_{ijk_{up}}$ can be computed independently and no need exists to recompute B when the value of N changes. This feature is very valuable when, for example, a prototype gravity model of a launch-region is desired but it is difficult to specify the order N required for an accurate representation; also note that N could be variable depending upon the frequency and amplitude of local gravity anomalies. Also, in case of a global gravity model, we can use a truncation of a spherical harmonic series for $\delta g_{up}(\ell)$ and compute

$$c_{ijk_{up}} = \int_D f_{ijk} \delta g_{up} \, dD \tag{30}$$

by carring out analytical integrations. This reduces Eq. (3) to a truncated Fourier series. Because a continuous spectrum of "measurements" is used, we can expect more accurate approximations.

6.1 Discussion

The recursive relationship of the orthonormal polynomials is very similar to the relationship previously used with Chebyshev polynomials. This formulation however does contain two additional division operations as well as a square root evaluation which must be performed on each polynomial. If one division and the square root values are saved during the preliminary calculation (SUBROUTINE ORTHO) and passed on for the actual evaluation, (SUBROUTINE MULT) a time savings can be realized at the small expense of a few (42) additional "words" of storage.

Note that even with this saving, the original Chebyshev polynomials can still be evaluated slightly faster, the difference only being the execution of a single division operation. However, when the advantages noted earlier are taken into account, the choice of orthonormal polynomials may be more beneficial, especially considering the effort that is involved with several different "set-ups" of variable polynomial orders and/or observation grid patterns where the Chebyshev version would require extensive computational time to process the least squares matrix and evaluate the coefficients. Indeed, the orthonormal approach does not require evaluation of $(A^TA)^{-1}$ and in addition, every coefficient c_{ijk} is independently computed.

7.0 Discussion of the Software

A variety of Fortran programs and subroutines were written during the course of this research effort. The software developed by Junkins and Saunders[2] was converted from its original CDC form to run on the VPI&SU IBM System 360 Model with 158 Processors. After correctly reproducing their test results, the software was modified to incorporate some programming efficiencies. From this starting point, new programs were developed to implement the orthonormal approach as well as test various additional approaches.

In general, the final software package can be divided into three separate stand-alone modules, regardless of the basic functions chosen.

Section I: A Mass Model 310 Initialization Section.

Section II: A Least Squares Reduction and Coefficient Determination Section.

Section III: A Test Section.

Section I, A Mass Model 310 Initialization Section, consists of a single fortran program, MASPOS, and its coded input data. This job step is identical for all the versions of the software developed so far and it will be required by any new version which makes use of Mass Model 310 to supply simulated gravity disturbances for use as observations or as a control to compare the modeling equation's calculations against. Clearly, this module can be replaced by input software if other sources (e.g., measurements) of gravity disturbances are available.

This program presently inputs coded values of the 1080 point masses

of Mass Model 310. Their location's are converted from the geodetic coordinates (H,λ,ϕ) to rectangular coordinates (x,y,z) and the mass codes are converted to their actual values. This information is then written onto a sequential access file (FILE 2), for use by SUBROUTINE PTMASS, which is described elsewhere. More specifically, PROGRAM MASPOS creates an unformatted sequential file, FILE 2, which contains the precomputed products of the gravitational constant and each point mass of Model 310 and the precomputed geocentric rectangular coordinates of each point mass. PROGRAM MASPOS requires as input coded information about Model 310 in the form of 30 cards. Each card represents 36 mass points in a grid row of equal latitude where the mass points are 50 longitude minutes apart. The mass points should be listed on each card by increasing longitude and the cards should be arranged in order of decreasing latitude. PROGRAM MASPOS reads only the coded multiplication factors for each of the 36 mass points of the grid row, where the coded factors indicate the mass of the respective mass points, $1 = -1.\times10^{19}$ grams, $2 = 0.$ grams, and $3 = +1.\times10^{19}$ grams. The coded factors are located in columns 25-60 of each data card. All other data card columns are ignored (for information only, columns 1-9 indicate south latitude in arc minutes of the grid row and columns 10-18 indicate east longitude in arc minutes of the west-most mass point in the row).

Section II, A Least Squares Reduction and Coefficient Determination Section, is where the actual modeling equation coefficients are calculated. Since the coefficients are determined by the choice of the basis functions and the choice of basis functions is a primary issue under study, each version of the software for Section II differs in several important aspects while the fundamental sections of the programs, such as establishment of

the finite element cells and grid pattern and determination of the gravity disturbances will be the same.

The main program for Section IIA, LOCALG, performs all the necessary preliminary calculations that are required by each version. Using card input, the finite element field is established and broken down into finite element cells and the observation grid pattern is also established. The critical field information is then written onto a storage file (FILE 3) for later use by PROGRAM FINEG. More specifically, PROGRAM LOCALG generates the A-matrix for a "typical" finite element cell and a set of gravity disturbance observations for each cell of a finite element gravity disturbance field. The gravity disturbance observations are evaluations of SUBROUTINE PTMASS on the uniform observation grid in each cell. The user defines a finite element gravity disturbance field by providing the following items as inputs to PROGRAM LOCALG:

1. The integral order of the locally valid polynomial approximating functions, NORDER, ($0 \leq$ NORDER ≤ 6).
2. The observation grid pattern (same for all cells) to include the number of observations in the up, eastern and northern directions, MOBSU, MOBSE, MOBSN. The total number of observations (MOBSU $\times$ MOBSE $\times$ MOBSN) should be at least three times the number of coefficients, NC (see Table 1), in the approximating model. Furthermore, the number of

observations in each direction should be one more than NORDER (MOBSU = MOBSE = MOBSN = NORDER + 1). The maximum number of observations allowed by the present program dimensions is 343 (a 7×7×7 observation grid). Non-uniform sample grids (e.g. 5×7×7, 6×5×4, etc.) have been found to produce unpredictable results - often an unacceptable fit.

3. The size of each finite element cell in H,λ,ϕ must be specified, HCELL, ALCELL, APCELL. HCELL is cell size in H (meters), (HCELL $\leq$ [HMAX - HMIN]). ALCELL is cell size in λ (degrees), (ALCELL $\leq$ [ALMAX - ALMIN]). APCELL is cell size in ϕ (degrees), (APCELL $\leq$ [APMAX - APMIN]).

4. The minimum and maximum H,λ,ϕ boundaries of the finite element field, HMIN, ALMIN, APMIN, HMAX, ALMAX, and APMAX. HMIN and HMAX are heights (H) above the reference ellipsoid in meters, (HMIN $\leq$ 0 and HMAX > HMIN). ALMIN and ALMAX are ellipsoidal longitudes (λ) in degrees, ($-180^\circ \leq$ ALMIN < ALMAX $\leq 180^\circ$). APMIN and APMAX are ellipsoidal latitudes (ϕ) in degrees, ($-90^\circ \leq$ APMIN < APMAX $\leq 90^\circ$).

These thirteen items are input on four data cards. NORDER, MOBSU, MOBSE, and MOBSN are input in (4I10) format on the first card. The remaining three cards must have a (3E20.14) format, with the finite

element field delimiters, HCELL, ALCELL, and APCELL on the second card, HMIN, ALMIN, and APMIN on the third card, and HMAX, ALMAX, and APMAX on the last card.

Because gravity disturbance observations are generated within PROGRAM LOCALG by SUBROUTINE PTMASS, PROGRAM LOCALG also requires as input the precomputed products of the gravitational constant and each of the mass points of Model 310 and the precomputed rectangular coordinates of each mass point. These quantities should have previously been stored sequentially via an unformatted write on FILE 2. PROGRAM LOCALG accesses these quantities with an unformatted read of FILE 2.

PROGRAM LOCALG produces as output an unformatted sequential file, FILE 3, which contains the A-matrix for a typical finite element cell and a set of gravity disturbance observations for each cell of the user specified finite element field. This file is properly formatted for later use by PROGRAM FINEG. PROGRAM LOCALG requires two supporting subroutines - SUBROUTINE CHEBY (or SUBROUTINE ORTHO) and SUBROUTINE PTMASS.

The basis polynomials are evaluated by the subroutines CHEBY or ORTHO depending on the particular version, and their outputs are used to construct the least squares matrix.

In particular, SUBROUTINE CHEBY (X, N, TA) returns Chebyshev polynomials in array TA, evaluated at a specified value, X, through the specified order, N. TA(1) = T_0(X), TA(2) = T_1(X), TA(3) = T_2(X), etc. TA is a seven element array; hence the specified order is bounded, $0 \leq N \leq 6$. SUBROUTINE ORTHO together with SUBROUTINE MULT perform the same function in the case of orthonormal polynomials.

In all versions, the gravity disturbance observations are determined for each cell by SUBROUTINE PTMASS, the final subroutine in Section IIA.

SUBROUTINE PTMASS is used both to produce observations for the least squares fitting process and to produce observations to test against for error analyses of finite element fields. Because SUBROUTINE PTMASS required the precomputed products of each point mass and the gravitational constant and the earth-fixed Cartesian coordinates of each point mass of Model 310, another small program, PROGRAM MASPOS, was necessary to calculate and store these quantities. SUBROUTINE PTMASS uses Mass Model 310 to determine the gravity disturbance observations at each cell grid point.

The rectangular coordinates (x,y,z) of the point are sent to the subroutine and the gravity disturbances are determined by:

$$\delta \underline{g} = \begin{Bmatrix} \delta g_x \\ \delta g_y \\ \delta g_z \end{Bmatrix} = \sum_{i=1}^{1080} \frac{PMVALS_i \underline{d}_i}{d_i^3} \tag{30}$$

where $PMVALS_i$ is the actual value of point mass i which was calculated by MASPOS in Section I

$$\underline{di} = \begin{Bmatrix} dx_i \\ dy_i \\ dz_i \end{Bmatrix} = \begin{Bmatrix} x - POSITS_{i_1} \\ y - POSITS_{i_2} \\ z - POSITS_{i_3} \end{Bmatrix} \tag{31}$$

$POSITS_{i_n}$ are the (x,y,z) coordinates of point mass i which was calculated by MASPOS in Section I and

PROGRAM LOCALG LOGIC FLOW (CHEBYSHEV VERSION)

START

INITIALIZE CONSTANTS

READ POINT MASS VALUES AND PRECOMPUTED POSITIONS ⟸ FILE 2

INPUT POLYNOMIAL ORDER, OBSERVATION GRID PATTERN, FINITE ELEMENT CELL SIZE, AND FINITE ELEMENT FIELD BOUNDS ⟸ NORDER, MOBSU, MOBSE, MOBSN, HCELL, ALCELL, APCELL, HMIN, ALMIN, APMIN, HMAX, ALMAX, APMAX

GENERATE ALL THE BASIS FUNCTION VALUES AT EACH OBSERVATION GRID POINT

WRITE CRITICAL FINITE ELEMENT FIELD INFORMATION ONTO THE SEQUENTIAL ACCESS FILE ⟹ FILE 3

GENERATE THE LEAST SQUARES MATRIX AND WRITE IT ONTO THE SEQUENTIAL ACCESS FILE ⟹ FILE 3

LOOP THROUGH THE FINITE ELEMENT FIELD, ONE CELL PER LOOP

GENERATE THE GRAVITY ANOMALY OBSERVATIONS (δg_{up}, δg_{east}, δg_{north}) WITH PTMASS

WRITE THE THREE OBSERVATION VECTORS ONTO THE SEQUENTIAL ACCESS FILE ⟹ FILE 3

GO TO THE NEXT FINITE ELEMENT CELL BY FIRST VARYING ϕ THEN λ AND FINALLY H UNTIL ALL THE CELLS HAVE BEEN OBSERVED

STOP

FIGURE 3

PROGRAM LOCALG LOGIC FLOW (ORTHONORMAL VERSION)

START

↓

INITIALIZE CONSTANTS

↓

READ POINT MASS VALUES AND PRECOMPUTED POSITIONS ⟸ FILE 2

↓

INPUT POLYNOMIAL ORDER, OBSERVATION GRID PATTERN, FINITE ELEMENT CELL SIZE AND FINITE ELEMENT FIELD BOUNDS ⟸ NORDER, MOBSU, MOBSE, MOBSN, HCELL, ALCELL, APCELL, HMIN, ALMIN, APMIN, HMAX, ALMAX, APMAX

↓

GENERATE THE COEFFICIENTS FOR THE BASIS FUNCTION IN THE H DIRECTION

↓

GENERATE ALL THE BASIS FUNCTION VALUES FOR EACH CONSTANT H PLANE

↓

GENERATE THE COEFFICIENTS FOR THE BASIC FUNCTION IN THE λ DIRECTION

↓

GENERATE ALL THE BASIS FUNCTION VALUES FOR EACH CONSTANT λ PLANE

↓

GENERATE THE COEFFICIENTS FOR THE BASIS FUNCTION IN THE ϕ DIRECTION

↓

GENERATE ALL THE BASIS FUNCTION VALUES FOR EACH CONSTANT ϕ PLANE

↓

WRITE THE CRITICAL FINITE ELEMENT FIELD INFORMATION AND ORTHONORMAL COEFFICIENTS ONTO THE SEQUENTIAL ACCESS FILE ⟹ FILE 3

↓

GENERATE THE LEAST SQUARES MATRIX AND WRITE IT ONTO THE SEQUENTIAL ACCESS FILE ⟹ FILE 3

↓

FIGURE 4

PROGRAM LOCALG LOGIC FLOW (ORTHONORMAL VERSION) CON'T

↓

LOOP THROUGH THE FINITE
ELEMENT FIELD, ONE CELL
PER LOOP

↓

GENERATE THE GRAVITY
ANOMALY OBSERVATIONS
($\delta g_{up}, \delta g_{east}, \delta g_{north}$)
WITH PTMASS

↓

WRITE THE THREE OBSERVATION
VECTORS ONTO THE SEQUENTIAL ⟹ FILE 3
ACCESS FILE

↓

GO TO THE NEXT FINITE ELE-
MENT CELL BY FIRST VARYING
ϕ THEN λ AND FINALLY H UNTIL
ALL THE CELLS HAVE BEEN
OBSERVED

↓

STOP

FIGURE 4 CON'T

$$d_i = dx_i^2 + dy_i^2 + dz_i^2$$

In particular, SUBROUTINE PTMASS produces gravity disturbance "observations", δg_{up}, δg_{east}, δg_{north}, to be used by PROGRAM LOCALG and PROGRAM FINEG for fitting purposes and to be used to test as data against SUBROUTINE FINITE. SUBROUTINE PTMASS produces the gravity disturbance data by using the 1080 point masses of Model 310. SUBROUTINE PTMASS requires the COMMON/MASPOS/ be filled with the precomputed products of the gravitational constant and each point mass and the geocentric rectangular coordinates of each point mass. SUBROUTINE PTMASS interfaces with the calling program via COMMON/XYZ/ through which the rectangular coordinates, x, y, z, of the calling point are received and the components of the gravity disturbance δg_x, δg_y, and δg_z (DELGX, DELGY, DELGZ) are transmitted. The rectangular components of the gravity disturbance must be transformed to geodetic components δg_{up}, δg_{east}, and δg_{north}, outside SUBROUTINE PTMASS (i.e., in the calling routine). The logic flow of PROGRAM LOCALG for the different versions are illustrated in Figures 3 and 4.

The Chebyshev and Orthonormal versions both use a second section, Section IIB to actually calculate the modeling coefficients. These two versions are different enough to warrant separate discussions.

In the Chebyshev version, the critical field information and the least squares matrix are read in. This version must calculate the modeling coefficients by solving the least squares solution. In lieu of solving the classical normal equations, a more efficient matrix reduction algorithm (ALSQ) is employed. A sequence of Householder elementary matrices are determined which, when premultiplied into the A-matrix

reduce it to a very particular upper triangular form. This triangular system can then be solved by backward substitution for the coefficients.

The A-matrix and its dimensions are passed to the main entry point of SUBROUTINE ALSQ. This section of ALSQ reduces the least squares matrix to its upper triangular form which is returned to FINEG. Next the gravity disturbance component vectors of each cell are read in one at a time. The reduced least squares matrix and the observation vector are then passed to the ALSQ1 entry point of SUBROUTINE ALSQ. This section operates on the observation vectors with the same sequence of Householder transformations, then solves the triangular system for the vectors of coefficients. The process is repeated for each of the up, east, and north measurement data to determine the corresponding coefficient vectors. The coefficients of each cell are written into separate records of a random-access file (FILE 3) to be used by SUBROUTINE FINITE in Section III. In particular, SUBROUTINE ALSQ (A,Y,B,R2,NN,MM,NA) solves the linear least squares problem, $||AB - Y||$ = minimum. SUBROUTINE ALSQ requires as input the coefficient matrix, A, the observation vector which is to fit, Y, the number of rows used in the A-matrix, NN, the number of columns used in the A-matrix, MM, and the first dimension of the A-matrix, NA. NA must be equal to one plus the maximum number of rows in the A-matrix ($NA = NN_{max} + 1$). SUBROUTINE ALSQ returns B, the coefficients of the fit and R2, the sum of the squares of the residuals.

The Orthonormal version of Section IIB produces exactly the same result but the process is much simplier. The simplification is based on the results found in section 5.0 of this report which shows that the

least squares coefficients are given by Eq. (29). Thus, the only processing needed, is to multiply the transpose of the unreduced least squares matrix (i.e., the A-matrix) by the observation vectors (one at a time) to produce the corresponding modeling coefficient vectors (one at a time). Since this was basically the job performed by the secondary entry point, ALSQ1 in the Chebyshev version, the new subroutine for the Orthonormal version is called ALSQ1. (It does not require a secondary entry point.) The coefficients are again returned to the main PROGRAM FINEG and written (in a set of three, one for each component) onto the appropriate record of the random access file (FILE 1). The logic flow charts of both of these versions are given in Figures 5 and 6. In particular, PROGRAM FINEG generates sets of locally valid coefficients C_{up}, C_{east}, and C_{north} for each cell of a finite element field which was previously specified by inputs to PROGRAM LOCALG. The only input required by PROGRAM FINEG is the unformatted sequential file, FILE 3, previously produced by PROGRAM LOCALG. PROGRAM FINEG produces as output a random access file, FILE 1, which contains, in effect, a finite element gravity disturbance field. The first record of the random access file contains certain important information about the finite element field (NORDER, boundaries, etc.). In addition to the first record, the random access file consists of one record of coefficients for each cell of the finite element field. Thus the number of records in the random access file will be equal to the number of cells in the finite element field plus 1. As indicated, PROGRAM FINEG requires one supporting subroutine - SUBROUTINE ALSQ.

PROGRAM FINEG LOGIC FLOW (CHEBYSHEV VERSION)

START

READ CRITICAL FINITE ELEMENT
FIELD INFORMATION FROM THE
SEQUENTIAL FILE

WRITE CRITICAL FINITE ELE-
MENT FIELD INFORMATION ONTO
THE FIRST RECORD OF THE
RANDOM ACCESS FILE

READ IN THE LEAST SQUARES
MATRIX

REDUCE THE LEAST SQUARES
MATRIX TO UPPER TRIANGULAR
FORM WITH SUBROUTINE ALSQ

LOOP THROUGH THE FINITE
ELEMENT FIELD ONE CELL
PER LOOP

READ GRAVITY ANOMALY
OBSERVATIONS FOR THIS CELL
FROM THE SEQUENTIAL FILE

COMPUTE POLYNOMIAL COEFFI-
CIENTS OF THIS FINITE ELE-
MENT MODEL WITH ENTRY ALSQ1

WRITE COEFFICIENTS OF THE
MODELING EQUATION ONTO THE
RANDOM ACCESS FILE

GO TO NEXT CELL

STOP

FIGURE 5

PROGRAM FINEG LOGIC FLOW (ORTHONORMAL VERSION)

START

↓

READ CRITICAL FINITE
ELEMENT FIELD INFORMATION
FROM THE SEQUENTIAL FILE

↓

WRITE CRITICAL FINITE ELEMENT
FIELD INFORMATION ONTO THE
FIRST RECORD OF THE RANDOM
ACCESS FILE

↓

READ IN THE LEAST SQUARES
MATRIX

↓

LOOP THROUGH THE FINITE
ELEMENT FIELD, ONE CELL
PER LOOP

↓

READ GRAVITY ANOMALY OB-
SERVATIONS FOR THIS CELL
FROM THE SEQUENTIAL FILE

↓

COMPUTE POLYNOMIAL COEFFI-
CIENTS OF THE FINITE ELEMENT
MODEL WITH SUBROUTINE ALSQ1

↓

WRITE COEFFICIENTS OF THE
MODELING EQUATION ONTO THE
RANDOM ACCESS FILE

↓

GO TO NEXT CELL

↓

STOP

FIGURE 6

Section III, Test Section, is the portion of the software system which evaluates the accuracy and speed of the new modeling approaches verses the Mass Model 310 technique. The main program of this section, FINITES, is the same for both versions of the software which have been developed thus far. The only changes that are necessary are the DIMENSION statements which declare the type and number of the critical field data that must be read from record number 1 (one) of the random access file (FILE 1), the READ statement which performs this operation and the common statements that transfer it to FINITE. Everything else is the same and the procedure is as follows.

After the critical field data are read in, the information produced in Section I for Mass Model 310 is also read in. This information is used to calculate the "actual" gravity disturbance measurements at each test point so that the modeling equation's values at the same points can be compared. Next, the intervals between test points (ISTEPH, ISTEPL, ISTEPP) are read in from cards along with the minimum (HMIN, ALMIN, APMIN) coordinates and maximum coordinates (HMAX, ALMAX, APMAX) of the test region. These coordinates are ellipsoidal. After the validity of the test pattern has been determined, the error analysis is initialized. The ellipsoidal coordinates (H, ALAM, APHI) of each test point are then passed to SUBROUTINE FINITE which returns the three gravity disturbance components which the modeling equation has calculated. These values are saved and the (x,y,z) coordinates of the same test point are determined and sent to SUBROUTINE PTMASS. This subroutine is identical to the subroutine PTMASS discussed earlier. It returns the three gravity disturbance components which Mass Model 310 has calculated. The

disturbance gravity components from PTMASS are first converted to their ellipsoidal values and the components of the two techniques are compared on a point-by-point basis; the error analysis is printed out (the means, standard deviations, and absolute value of maximum errors are printed out along with the average time required to calculate each point).

As noted previously, the actual evaluation of the gravitational disturbances is performed by subroutine PTMASS. For the Chebyshev and Orthonormal versions, although the basis functions are different, the procedure is exactly the same. First the ellipsoidal coordinates (H, ALAM, APHI) of the test point are received from the main program in the common block labelled HLP, and the point is compared against the modeled region minimum (HMIN, ALMIN, APMIN) and maximum (HMAX, ALMAX, APMAX) to see if the test point lies within the region modeled in Section II. If not, the gravity disturbance is evaluated by a call of SUBROUTINE PTMASS, a message indicating this fact is printed and the PTMASS values are returned to FINTES. If the point does lie within the modeled region, the critical field information is used to determine which finite element cell contains the test point. When the cell is determined, the three sets of coefficients (CU,CE,CN) are read from the random access file (FILE 1). Next the coordinates of the test point is normalized with respect to the finite element cell, and these values (X_1,X_2,X_3) are then passed to SUBROUTINE CHEBY for the Chebyshev version or SUBROUTINE MULT for the Orthonormal version and the basis functions are evaluated at that point. These subroutines are exactly the same as the ones used in Section IIA, to evaluate the basis functions to determine the least squares matrix. The gravity disturbance components are then determined as the sum of the products of the coefficients multiplied by the

evaluated basis function terms. These values are then returned to FINTES for error analysis.

In particular, PROGRAM FINTES is a simple error analysis routine. It compares the gravity disturbance as approximated by SUBROUTINE FINITE with the gravity disturbance as evaluated by SUBROUTINE PTMASS (Model 310). Residuals are printed for each point of a user specified grid, along with the mean of the residuals, the root mean square of the residuals, and the maximum absolute value of the residuals, for each component of the gravity disturbance. The following items are required as input to specify the sample grid:

1. The number of samples in each direction, ISTEPH, ISTEPL, and ISTEPP, (ISTEPH > 0, ISTEPL > 0, and ISTEPP > 0). The total number of sample points on the grid will be the product of these three numbers.
2. The minimum sample grid coordinate values ("the lower corner"), HMIN (meters), ALMIN (degrees), and APMIN (degrees) and the maximum sample grid coordinate values ("the upper corner"), HMAX (meters), ALMAX (degrees), and APMAX (degrees), ($0 \leq$ HMIN $\leq$ HMAX, $-180^{\circ} \leq$ ALMIN $\leq$ ALMAX $\leq 180^{\circ}$, $-90^{\circ} \leq$ APMIN $\leq$ APMAX $\leq 90^{\circ}$).

These nine items are input on three data cards. ISTEPH, ISTEPL, and ISTEPP are input on the first data card in a (3I5) format. HMIN, ALMIN, and APMIN are input on the second card in (F10.0, 2F10.2) format and HMAX, ALMAX, and APMAX are input on the third card in the same format.

PROGRAM FINTES requires three supporting subroutines - SUBROUTINE FINITE, SUBROUTINE CHEBY or ORTHO (see discussion of these routines above), and SUBROUTINE PTMASS (also discussed above).

Because PROGRAM FINTES uses SUBROUTINE PTMASS, it also requires as input the unformatted sequential file, FILE 2, containing the precomputed products of the gravitational constant and each point mass of Model 310 and the precomputed rectangular coordinates of each point mass.

SUBROUTINE FINITE directly replaces Model 310 in applications involving the gravity disturbance $\delta \underline{g}$. Given the ellipsoidal coordinates, H, ALAM, and APHI (H,λ,ϕ), of a specified point, SUBROUTINE FINITE returns the ellipsoidal components, GU, GE, and GN $(\delta G_{up}, \delta G_{east}, \delta G_{north})$ of the gravity disturbance approximation, $\underline{\delta G}$. SUBROUTINE FINITE determines which set of finite element coefficients to use at the specified point, reads them into core, and computes $\underline{\delta G}$. SUBROUTINE FINITE requires SUBROUTINE CHEBY or ORTHO (described previously) to produce basis polynomials at the specified point. Whenever the specified point does not lie within the bounds of the finite element field currently accessed, SUBROUTINE FINITE calls SUBROUTINE PTMASS, writes an error message, and sets the error flag ISITIN to zero.

Prior to the first call to SUBROUTINE FINITE, the random access file, FILE 1, must be made available to the calling program, and two flags, IFLAG and ISITIN, must be set to zero and included in a COMMON in both the calling program and SUBROUTINE FINITE. This could be COMMON /HLP/ if necessary (COMMON /HLP/ is the vehicle by which SUBROUTINE FINITE receives the calling coordinates and returns the components of $\underline{\delta G}$), although COMMON /IMARK/ was used.

The following remarks are in order when (1)C FORTRAN is used. The requirements of CDC FORTRAN dictated that the CDC utility subroutines,

subroutines, OPENMS, WRITMS, READMS, and CLOSMS, be used to access the random access file. Furthermore, these CDC utility routines require that the user establish an index array (IMARK, in this case) for the random access file. If an index array is required by a particular version of FORTRAN, the index array, IMARK, should be included in a COMMON in both the calling program and SUBROUTINE FINITE. The logic flow for SUBROUTINE FINITE is displayed in Figure 7.

8.0 Tradeoff Studies

Several tradeoff studies were conducted on the VPI&SU IBM System 360 Model with 158 Processors. The central processor execution times of SUBROUTINE FINITE were compared to SUBROUTINE PTMASS (Model 310). Although it is really a price that is paid only once and a priori in the laboratory, the central processor execution times necessary to create finite element fields were examined. Consideration was given to the tradeoff: total number of finite element coefficients versus NORDER versus cell size, for finite element models of a given region. Lastly, error analyses for a variety of NORDER's and cell sizes were conducted; specifically, the maximum absolute error versus NORDER versus cell size tradeoff was examined.

Table 3 shows the execution time test results for three models: Chebyshev, Orthonormal, and Pointmass. SUBROUTINE PTMASS has a nearly constant run time. The run time for SUBROUTINE FINITE, however, depends on NORDER, the basis polynomial order, and on whether or not a random access call is needed. A random access call is needed anytime the set of coefficients currently in core is not the set for the cell in which the calling coordinates lie; that is, the previous calling coordinates were in a different finite element cell than the current calling coor-

SUBROUTINE FINITE LOGIC FLOW

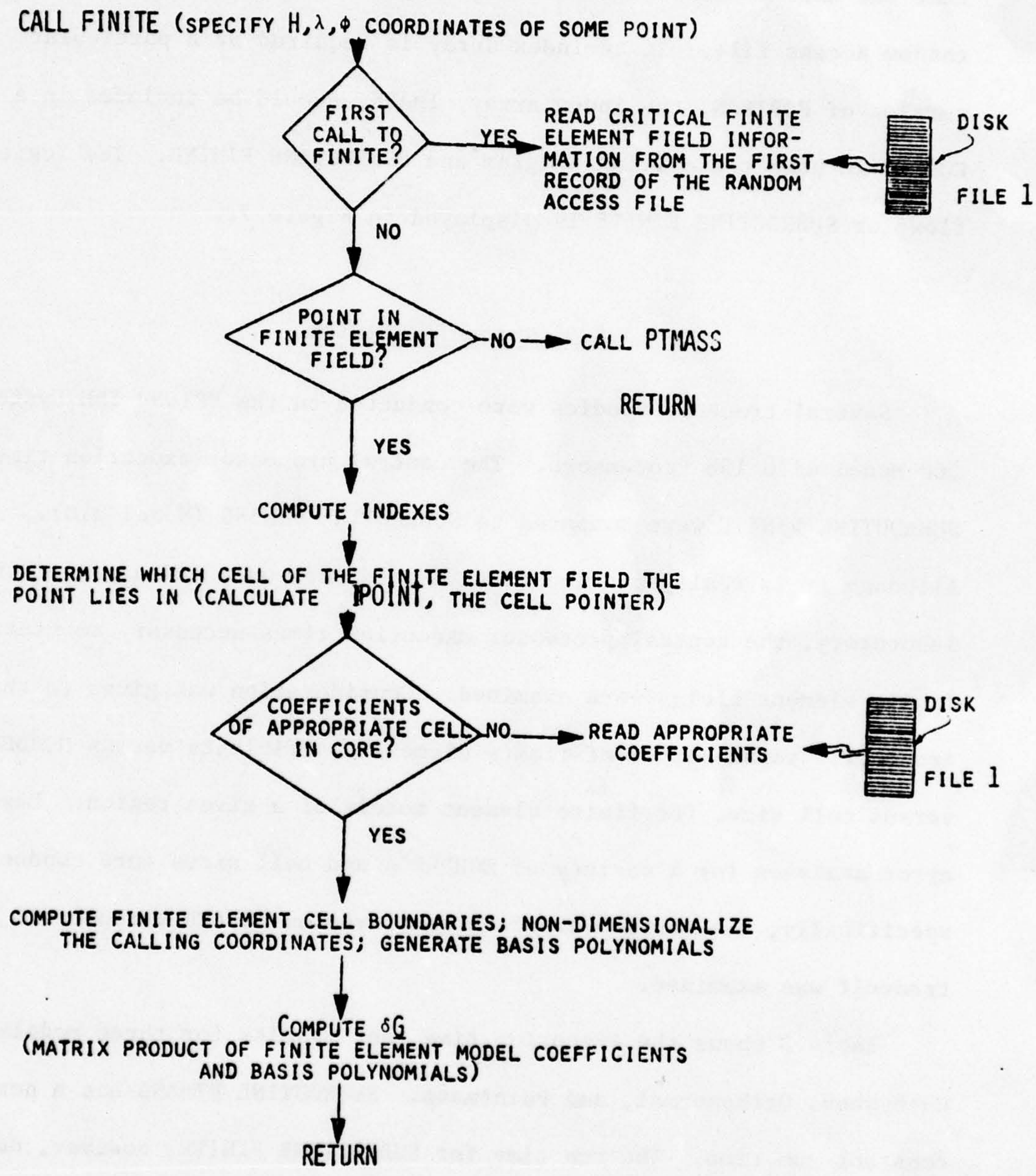


FIGURE 7 SUBROUTINE FINITE LOGIC FLOW

Table 3 Central Processor Execution Time Comparison: Model 310 vs. Chebyshev Polynomial Model vs. Orthonormal Polynomial Model (average time per point).

N	Chebyshev	Orthonormal	Pointmass
3	2.2 msec	2.5 msec	230 msec
4	3.0	3.3	230
5	3.9	4.5	230
6	5.4	6.0	230
3	1/105	1/92	
4	1/77	1/70	
5	1/59	1/51	
6	1/43	1/38	

dinates. The execution time ratios show that even for the worst cases the Orthonormal model and the Chebyshev model will be at least 40 times faster than an explicit point mass disturbance acceleration model such as Model 310. Note that by using the recurrence relations (24) we can reduce the execution times for Orthonormal so that they are almost identical to the ones produced by Chebyshev.

A certain price must be paid to generate finite element fields. Execution times are shown in Table 4. The price per cell must be multiplied by the total number of cells in the finite element field and added to the price to generate the numbers $f_{ijk}(\ell)$. Note that over 90% of the price paid per cell is just the generation of the gravity observations from Model 310. Note the significant time-savings in using the orthonormal model.

The total number of coefficients needed for various finite element field models of the $35^\circ \times 40^\circ$ region covered by Model 310 are shown in Table 5. Table 5 assumes that H_{cell} will be constant at 300 km and that 300 km will be the maximum altitude to be modeled. For a given cell size, it shows how many finite element cells would be needed to model the region and how many total coefficients would be needed for various NORDERS to model the region. Based upon error analyses conducted on a small volume near the center of the $35^\circ \times 40^\circ$ region, expected maximum error bounds were determined for certain cell sizes and NORDERS. The upper solid and dashed underline indicates an expected maximum absolute error in mgals. Solid underlines in both cases indicate that the expected maximum absolute error was verified for small test volumes while dashes indicate the apparent trend.

In order to study the maximum absolute error as functions of NORDER and cell size, it was decided to consider the maximum absolute error as

Table 4

Central Processor Times to Create Finite Element Fields

	Chebyshev				Orthonormal			
N:	3	4	5	6	3	4	5	6
Time to generate matrix A (sec)	.32	.61	1.35	2.90	.41	.70	1.57	3.16
Time to generate the observations using Model 310 (per cell)	14.87	29.27	50.82	80.12	14.48	28.63	50.43	79.65
Time to process matrix A	1.24	5.45	22.20	77.39	.58	.80	1.29	2.43
Time to calculate the coefficients (per cell)	.43	1.43	3.98	9.23	.22	.75	2.00	5.07
Total Prices								
Time to generate and process matrix A	1.56	6.06	23.55	80.29	.99	1.50	2.86	5.59
Time to generate observations and coefficients using Model 310 (per cell)	15.30	30.70	54.80	89.35	14.70	29.38	52.43	84.72

Table 5 Total number of Coefficients vs. N. vs. Cell Size For a 300 km × 35° × 40° Finite Element Field. (Chebyshev and Orthonormal)

Cell Size (λ_{cell} × ϕ_{cell})	No. of Cells	3	4	5	6
1.0° × 1.0°	35×40 = 1400	84,000	147,000	235,200	352,800
1.25° × 1.25°	28×32 = 896	53,760	94,080	150,528	225,792
1.5° × 1.5°	24×27 = 648	38,880	68,040	108,864	163,296*
1.75° × 1.75°	20×23 = 460	27,600	48,300	77,280	115,920
2.0° × 2.0°	18×20 = 360	21,600	37,800	60,480	90,720†
2.25° × 2.25°	16×18 = 288	17,280	30,240	48,384	72,576
2.5° × 2.5°	14×16 = 224	13,440	23,520	37,632	56,448
2.75° × 2.75°	13×15 = 195	11,700	20,475	32,760	49,140
3.0° × 3.0°	12×14 = 168	10,080	17,640	28,224	42,336

Given: H_{Cell} = constant = 300 km

* = Finite element fields above this line are expected to have $|error|_{max} \leq 1.5$ mgals.

† = Finite element fields above this line are expected to have $|error|_{max} \leq 3.0$ mgals.

a function of cell size only for fixed NORDER (see Figure 8) and to consider maximum absolute error as a function of NORDER for fixed cell sizes (see Figure 9). For all cases, cell altitude (H_{cell}) was fixed at 300 km.

Figure 8 shows that the maximum absolute error varies approximately as the square of the cell dimension in λ or ϕ. For constant cell altitude and for small λ_{cell} and ϕ, the volume will vary approximately as the square of λ_{cell} of ϕ_{cell}, if $\lambda_{cell} = \phi_{cell}$. Hence the maximum absolute error is really proportional to cell volume. Figure 9 shows that the maximum absolute error varies approximately inversely as the square of NORDER. As the number of coefficients per cell varies approximately as the square of NORDER, the maximum absolute error may be considered to vary nearly inversely with the number of coefficients per cell.

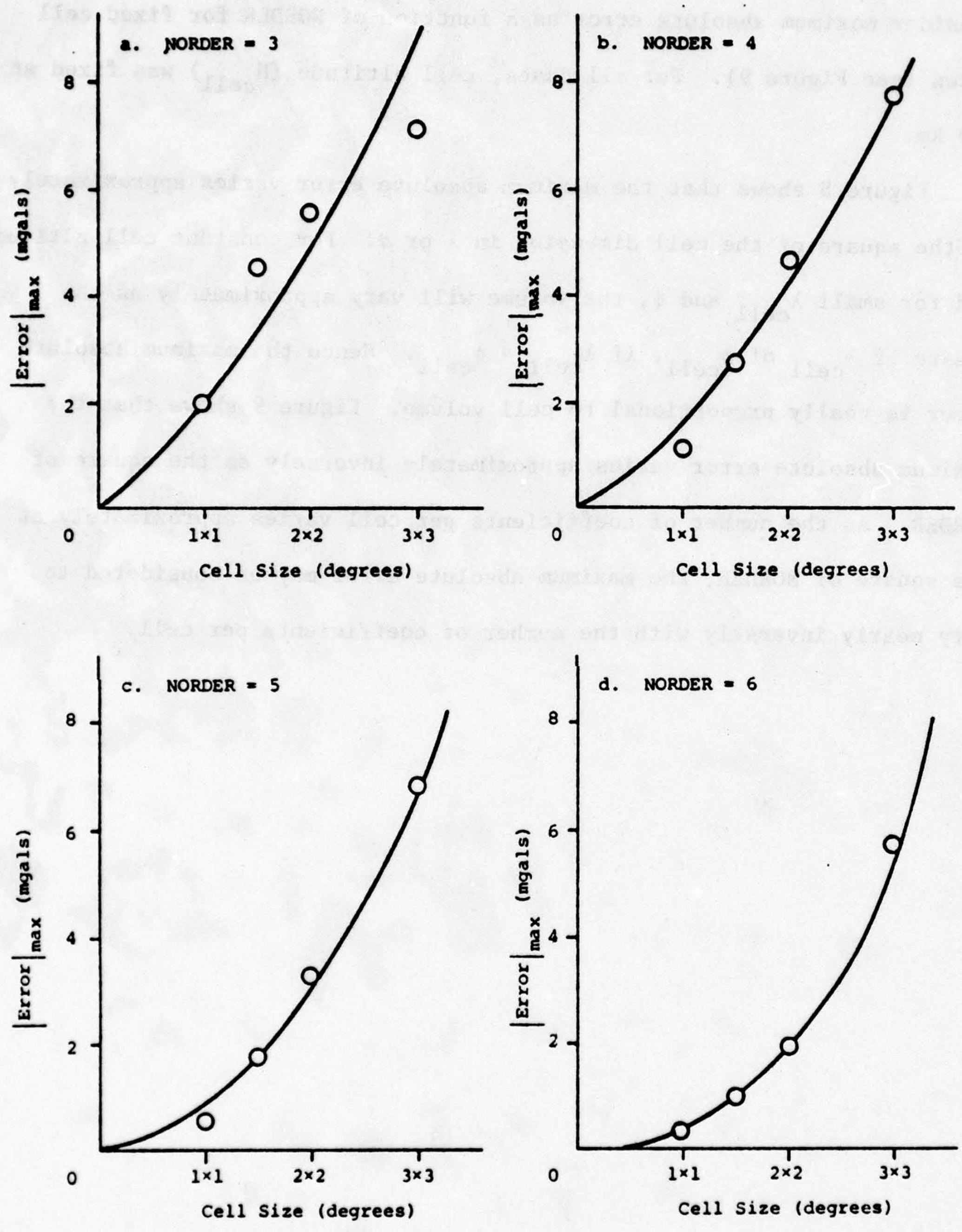


Figure 8 Maximum Absolute Error versus Cell Size for Fixed NORDER's.

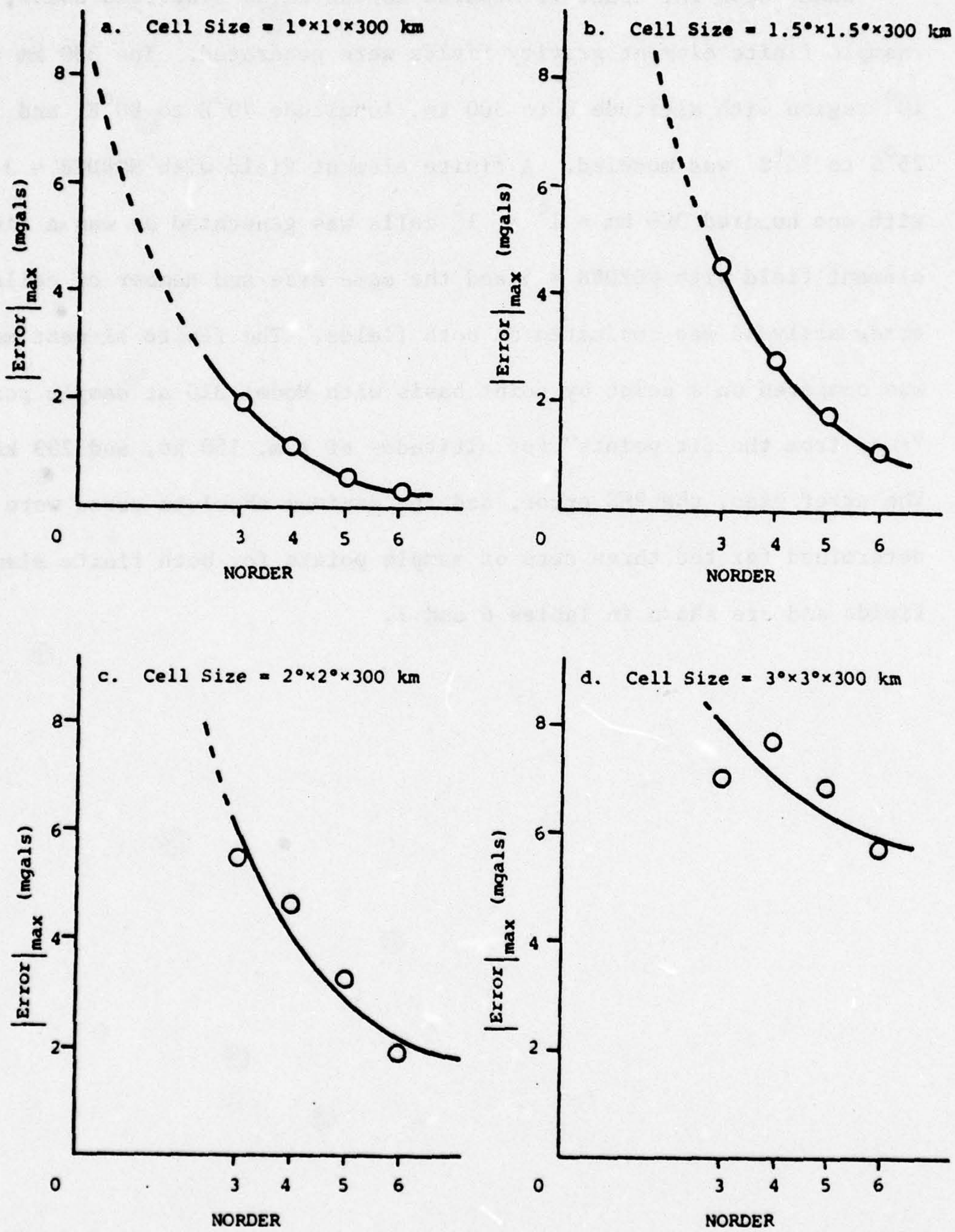


Figure 9 Maximum Absolute Error versus NORDER for Fixed Cell Sizes.

9.0 Example Finite Element Gravity Models

Based upon the tradeoff studies conducted as described above, two example finite element gravity fields were generated. The 300 km × 10° × 10° region with altitude 0 to 300 km, longitude 70°E to 80°E, and latitude 25°S to 35°S was modeled. A finite element field with NORDER = 3 and with one hundred 300 km × 1° × 1° cells was generated as was a finite element field with NORDER = 5 and the same size and number of cells. An error analysis was conducted on both fields. The finite element model was compared on a point by point basis with Model 310 at sample points "away from the fit points" for altitudes of 1 m, 150 km, and 299 km. The error mean, the RMS error, and the maximum absolute error were determined for the three sets of sample points for both finite element fields and are shown in Tables 6 and 7.

Table 6

N=3 Example Finite Element Gravity Disturbance Field
(Chebyshev and Orthonormal)

	δg_{up} (mgals)	δg_{east} (mgals)	δg_{north} (mgals)
H = 1 m			
$\overline{\text{ERROR}}$	-.005	-.001	.004
RMS ERROR	.963	.645	.727
$\lvert\text{ERROR}\rvert_{MAX}$	2.717	2.074	2.236
H = 150 km			
$\overline{\text{ERROR}}$	-.023	.003	.009
RMS ERROR	.352	.224	.270
$\lvert\text{ERROR}\rvert_{MAX}$	1.149	.701	1.039
H = 299 km			
$\overline{\text{ERROR}}$	.018	-.006	-.007
RMS ERROR	.424	.277	.322
$\lvert\text{ERROR}\rvert_{MAX}$	1.130	.771	1.002

Table 7

N=5 Example Finite Element Gravity Disturbance Field
(Chebyshev and Orthonormal)

	δg_{up} (mgals)	δg_{east} (mgals)	δg_{north} (mgals)
H = 1 m			
$\overline{\text{ERROR}}$	-.004	.001	.002
RMS ERROR	.224	.150	.167
$\lvert\text{ERROR}\rvert_{MAX}$	.918	.637	.581
H = 150 km			
$\overline{\text{ERROR}}$	.007	-.002	-.004
RMS ERROR	.071	.044	.055
$\lvert\text{ERROR}\rvert_{MAX}$	.216	.157	.215
H = 299 km			
$\overline{\text{ERROR}}$	-.002	.001	.001
RMS ERROR	.101	.067	.077
$\lvert\text{ERROR}\rvert_{MAX}$	.329	.254	.249

10.0 Orthogonal Approach

The main purpose of this section is to indicate a way to produce an accurate model for the geopotential itself. The Chebyshev and Orthonormal approaches model the gravity disturbance acceleration components independently and do not take into account the fundamental functional relationship

$$\underline{g} = \nabla U \tag{32}$$

Therefore, no rigorous representation for U can be obtained by using gravity components derived either by the Chebyshev or Orthonormal approach.

A finite element procedure for modeling the geopotential that takes the functional relationships (32) between the geopotential and its gradient into account appears desirable. Not only will this enable us to obtain an approximation for U, but a significant reduction in the overall cost of determining gradient models is anticipated.

Let us consider the following preliminary approximation of the geopotential disturbance function

$$\delta U = \sum_{n=0}^{N} \sum_{i=0}^{n} \sum_{j=0}^{i-n} c_{ijk}\, f_{ijk} \quad , \quad k = n-i-j \tag{33}$$

where c_{ijk} are modeling constants to be determined, and $f_{ijk}(x,y,z)$ are arbitrary basis functions. Using Eq. (32) and (33) we can write the acceleration disturbances as follows

$$\delta g_x = \sum_{n=0}^{N} \sum_{i=0}^{n} \sum_{j=0}^{i-n} c_{ijk}\, f_{ijk}^{x} \tag{34a}$$

$$\delta g_y = \sum_{n=0}^{N} \sum_{i=0}^{n} \sum_{j=0}^{i-n} c_{ijk} \, f_{ijk}^{y} \tag{34b}$$

$$\delta g_z = \sum_{n=0}^{N} \sum_{i=0}^{n} \sum_{j=0}^{i-n} c_{ijk} \, f_{ijk}^{z} \tag{34c}$$

with

$$f_{ijk}^{x} = \frac{\partial}{\partial x} f_{ijk}, \quad f_{ijk}^{y} = \frac{\partial}{\partial y} f_{ijk}, \quad f_{ijk}^{z} = \frac{\partial}{\partial z} f_{ijk} \tag{35}$$

The local modeling coefficients c_{ijk} are to be determined via a least squares approximation technique, taking into account the local observations of all three gravity components ($\delta g_x, \delta g_y, \delta g_z$) simultaneously.

The normal equations can be written in the usual manner

$$\underline{c} = (A^T A)^{-1} A^T \delta \underline{g}^* \tag{36}$$

where

$$\underline{c} = \begin{Bmatrix} c_{000} \\ c_{001} \\ c_{010} \\ \cdots\cdot \\ c_{N00} \end{Bmatrix}, \quad \delta \underline{g}^* = \begin{Bmatrix} \delta g_x^*(1) \\ \delta g_y^*(1) \\ \delta g_z^*(1) \\ \cdots\cdot \\ \delta g_x^*(m) \\ \delta g_y^*(m) \\ \delta g_z^*(m) \end{Bmatrix} \tag{37}$$

are the column matrix of modeling constants c_{ijk} and the column matrix of gravity disturbance observations, respectively. Furthermore,

$$A = \begin{bmatrix} f^x_{000}(1) & f^x_{001}(1) & f^x_{010}(1) & \cdots & f^x_{N00}(1) \\ f^y_{000}(1) & f^y_{001}(1) & f^y_{010}(1) & \cdots & f^y_{N00}(1) \\ f^z_{000}(1) & f^z_{001}(1) & f^z_{010}(1) & \cdots & f^z_{N00}(1) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f^x_{000}(m) & f^x_{001}(m) & f^x_{010}(m) & \cdots & f^x_{N00}(m) \\ f^y_{000}(m) & f^y_{001}(m) & f^y_{010}(m) & \cdots & f^y_{N00}(m) \\ f^z_{000}(m) & f^z_{001}(m) & f^z_{010}(m) & \cdots & f^z_{N00}(m) \end{bmatrix} \tag{38}$$

in which m indicates the total number of local measurements of the disturbance acceleration.

It is now possible to evaluate the constants c_{ijk} from Eq. (36) in the same manner as the Chebyshev approach. However, we wish to introduce a significant simplification by using orthogonal basis functions, similar to the Orthonormal approach. To this end, let us write the expression for a generic element of the matrix A^TA as follows

$$\sum_{\ell=1}^{m} F_{ijk,\alpha\beta\gamma}(\ell) \tag{39}$$

where

$$F_{ijk,\alpha\beta\gamma}(\ell) = f^x_{ijk}(\ell)\, f^x_{\alpha\beta\gamma}(\ell) + f^y_{ijk}(\ell)\, f^y_{\alpha\beta\gamma}(\ell) + f^z_{ijk}(\ell)\, f^z_{\alpha\beta\gamma}(\ell) \tag{40}$$

Let us now introduce basis functions of the form

$$f_{ijk}(x,y,z) = f_i(x)\, f_j(y)\, f_k(z) \tag{41}$$

so that,

$$f^x_{ijk} = f'_i f_j f_k \quad , \quad f^y_{ijk} = f_i f'_j f_k \quad , \quad f^z_{ijk} = f_i f_j f'_k \tag{42}$$

where the derivatives denoted by primes are taken with respect to the argument. It is clear that the generic element (39) can be written as

$$
\begin{aligned}
&\sum_x f_i' f_\alpha' \sum_y f_j f_\beta \sum_z f_k f_\alpha \\
+\ &\sum_x f_i f_\alpha \sum_y f_j' f_\beta' \sum_z f_k f_\alpha \qquad\qquad (43)\\
+\ &\sum_x f_i f_\alpha \sum_y f_j f_\beta \sum_z f_k' f_\alpha'
\end{aligned}
$$

where the respective summations run over all the observations in the respective directions.

Next, let us assume that the sets of functions

$$\{f_i\}\ ,\ \{f_j\}\ ,\ \{f_k\}$$

are orthonormal over the interval of validity and that similarly

$$\{f_i'\}\ ,\ \{f_j'\}\ ,\ \{f_k'\}$$

are orthogonal with respect to the same interval, then it is clear that $A^T A = D$ becomes diagonal with diagonal elements D_{ijk} given by

$$D_{ijk} = \sum_x f_i'^2 + \sum_y f_j'^2 + \sum_z f_k'^2$$

Equation (36) becomes

$$\underline{c} = D^{-1} A^T \ \delta \underline{g}^*$$

or, more explicitly,

$$c_{ijk} = D^{-1}_{ijk} \sum_{\ell=1}^{m} \left(f^{x}_{ijk} \ \delta g^*_x + f^{y}_{ijk} \ \delta g^*_y + f^{z}_{ijk} \ \delta g^*_z \right)_\ell \qquad (44)$$

The only remaining task is to find an orthonormal set of functions $\{f_i(x)\}$ such that the derivative set $\{f_i(x)\}$ is also orthogonal over the

same interval of validity. Note that the same set can be used in all three directions x,y,z.

To date, only NORDER = 3 has been tried. As expected, however, not enough accuracy is obtained and further work is underway for higher NORDERs. It is expected that the number of modeling constants c_{ijk} will be significantly reduced compared to the total number of modeling constants c_{ijk}^{ϕ}, c_{ijk}^{λ}, c_{ijk}^{γ}, necessary to model the gravity disturbance components independently. This fact, not only increases the computational speed of the algorithm, but also reduces the storage requirements. Moreover, it is now possible to obtain a valid approximation for the geopotential itself.

11.0 References

1. Junkins, J. L., "Development of Finite Element Models for the Earth's Gravity Field, Phase I: Macro Gravity Model for the Satellite Orbit Integration," University of Virginia, RLES Report No. UVA/525023/ESS77/103, February 1977.

2. Junkins, J. L., Saunders, J. T., "Development of Finite Element Models for the Earth's Gravity Field, Phase II: Fine Structure Disturbance Gravity Representations," University of Virginia, RLES Report No. UVA/525023/ESS77/104, March 1977.

3. Rainville, E. D., Special Functions, The MacMillan Company, New York, 1960, p. 151.

12.0 Software Listings

---

*These Orthonormal programs and subroutines represent minor modifications of the corresponding Chebyshev Polynomial Software.

**The numerical results (for fixed NORDERS) are identical for the two approaches. The orthonormal vs. Chebyshev tradeoffs are with regard to efficiency, not accuracy, see the above discussion in Section 8.0.

MODEL 310

POINT MASS DISTURBANCE ACCELERATION CALCULATION

```
C     PROGRAM MASPOS(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE2)        *
C-----------------------------------------------------------------------
C   SECTION 1
C     LOCAL GRAVITY MODEL -- MASS MODEL 310 INITIALIZATION
C
C     BY                JOHN L. JUNKINS AND JOHN SAUNDERS
C
C     MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C             DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C             SCHOOL OF ENGINEERING
C             VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C             BLACKSBURG, VIRGINIA   24061
C
C     DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C-----------------------------------------------------------------------
C
C    FOR P,  SOME POINT IN SPACE
C    X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES
C    H,ALAM,APHI--ELLIPSOIDAL COORDINATES
C    RNI--EAST-WEST RADIUS OF CURVATURE
C    H--HEIGHT ABOVE REF. ELLIPSOID ALONG NORMAL TO P
C    ALAM--ANGLE LAMBDA, GEODETIC/GEOCENTRIC LONGITUDE (RADIANS) OF P
C    APHI--ANGLE PHI, GEODETIC LATITUDE (RADIANS) OF P
C
C    INPUTS
C
C    FROM CARDS
C    FORMAT 903
C    MASSES--CODED VALUES WHICH REPRESENT THE MASS OF EACH OF THE 1080
C       MASS POINTS
C
C    THE CARD INPUT FOR THIS SAMPLE RUN IS AS FOLLOWS:
C         925      3625         333333121133111331133321113111111313
C         975      3625         311131131131221111311111113113111131
C        1025      3625         111313111133113131311111321113333333
C        1075      3625         313331333311133313123233113111113111
C        1125      3625         111333313313113133111333333231111133
C        1175      3625         131111233113333333333233333333333333
C        1225      3625         131323131113313211333311331111312333
C        1275      3625         113111233111313131133131113111332111
C        1325      3625         133131131111233113311133311331313311
C        1375      3625         111131131111131132331133312313313311
C        1425      3625         133331333332123131121331321311113131
C        1475      3625         333323333333232133112311333333111333
C        1525      3625         312313333313311131111331133131113133
C        1575      3625         231111311131131131111111131111313131
C        1625      3625         113112111133323131333311331313133333
C        1675      3625         313313333311333331231322111113111133
C        1725      3625         133333331313331113133333331312333111
C        1775      3625         332111111111332113133231123313111131
C        1825      3625         113133131333111113121323133331131113
C        1875      3625         323333111331113331131333133333111311
C        1925      3625         133133331111312311313333111333113111
C        1975      3625         111113132133133131133111313313313333
```

```
      C      2025       3625        131331133311332111112333211213331113
      C      2075       3625        113123332113333333121111111111333333
      C      2125       3625        311133332313333311311111113131112313
      C      2175       3625        111311131311311331331311111311233331
      C      2225       3625        223111331333331333333111311111131133
      C      2275       3625        311313331333331331331311311231111133
      C      2325       3625        311233111312113111111231333332312111
      C      2375       3625        333323111111121211333323333333231331
      C
      C   PROCESS
      C
      C      THIS PROGRAM ACCEPTS CARD INPUT OF CODED VALUES WHICH REPRESENT
      C      THE 1080 POINT MASSES OF MASS MODEL 310.  THE ACTUAL MASS VALUES
      C      ARE CALCULATED AND THE POINT COORDINATES ARE CONVERTED FROM THEIR
      C      ELLIPSOIDAL COORDINATES (H,LAMDA,PHI) TO THEIR RECTANGULAR CO-
      C      ORDINATES (X,Y,Z).  THE MASS AND RECTANGULAR COORDINATES OF EACH
      C      POINT IS THEN WRITTEN ONTO A SEQUENTIAL ACCESS FILE (FILE 2).
      C
      C   OUTPUTS
      C
      C   TO DISK
      C   FILE 2
      C   PMVALS--PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL CONSTANT AND THE
      C      1080 POINT MASSES (-1.E19, 0., OR +1.E19)
      C   POSITS--PRECOMPUTED EARTH-FIXED X,Y,Z COORDINATES OF THE 1080 POINT
      C      MASSES
      C
      C      THE CREATED FILE 2 IS USED BY SUBROUTINE PTMASS IN TWO LATER
      C      SECTIONS OF THIS SYSTEM OF PROGRAMS.  IN SECTION II(A), PTMASS
      C      USES MODEL 310 TO CALCULATE THE 'OBSERVATIONS' OF GRAVITY
      C      ANOMALIES AT SEVERAL GRID POINTS IN A FINITE ELEMENT CELL.  THESE
      C      OBSERVATIONS ARE NECESSARY TO DETERMINE THE COEFFICIENTS FOR THE
      C      MODELING EQUATION.  IN SECTION III, PTMASS USES MODEL 310 TO
      C      CALCULATE THE 'ACTUAL' GRAVITY ANOMALIES AT SPECIFIC POINTS.  THE
      C      'ACTUAL' VALUES ARE THEN COMPARED TO VALUES CALCULATED BY THE
      C      DETERMINED MODELING EQUATION FOR ERROR ANALYSIS.
      C
      C--------------------------------------------------------------------------
      C
0001         IMPLICIT REAL*8 (A-H,O-Z)
0002         DIMENSION  MASSES(1080),PMVALS(1080),POSITS(1080,3)
      C
0003         DATA A,B/6378160.D0,6356774.504D0/
0004         DATA E1GG/6.67D-14/
      C
0005     901 FORMAT(1H1,3X,9HMASSES(I),18X,1HX,19X,1HY,19X,1HZ)
0006     902 FORMAT(1X,4(3X,17(1H-)))
0007     903 FORMAT(24X,36I1,20X)
0008     905 FORMAT(1X,4(1PE20.10))
      C
      C   SET UP CONVERSION FACTOR FOR MINUTES OF DEGREE TO RADIANS
      C
0009         PI=DARCOS(-1.D0)
0010         RADMIN=10800.D0/PI
      C
```

```
                C   READ IN THE MASS CODES
                C
0011                READ(5,903) MASSES
                C
                C   FROM A STARTING POINT OF:
                C             H = -8000 KM.
                C       LAMDA =  3575'
                C         PHI =   875'
                C   CALCULATE THE POSITION OF THE 1080 MASSES
                C
0012                A2=A*A
0013                B2=B*B
0014                RHI=-80000.D0
0015                IPT=0
0016                PHIM=-875.D0
0017                DO 20 I=1,30
0018                PHIM=PHIM-50.D0
0019                PHI=PHIM/RADMIN
0020                SINP=DSIN(PHI)
0021                COSP=DCOS(PHI)
                C
0022                RN1=A2/DSQRT(A2*COSP*COSP + B2*SINP*SINP)
0023                TEMP=(RN1+RHI)*COSP
0024                TEMP2=B2*RN1/A2+RHI
0025                ALAMBM=3575.D0
0026                DO 20 J=1,36
0027                ALAMBM=ALAMBM+50.D0
0028                ALAMB=ALAMBM/RADMIN
0029                SINL=DSIN(ALAMB)
0030                COSL=DCOS(ALAMB)
                C
0031                IPT=IPT+1
0032                MI=MASSES(IPT)-2
0033                RMI=MI
                C
                C   USING THE INPUT DATA CALCULATE EACH MASS AND STORE ITS VALUE
                C
0034                PMVALS(IPT)=RMI*BIGG*1.D19
                C
                C   STORE THE POSITION COORDINATES IN THE MATRIX POSITS
                C
0035                POSITS(IPT,1)=TEMP*COSL
0036                POSITS(IPT,2)=TEMP*SINL
0037                POSITS(IPT,3)=TEMP2*SINP
0038             20 CONTINUE
                C
                C   WRITE MASS VALUES AND X,Y,Z COORDINATES ONTO FILE 2
                C
0039                WRITE(2) PMVALS,POSITS
                C
0040                IPT=0
                C
                C   OUTPUT THE ARRAYS OF MASS VALUES AND X,Y,Z COORDINATES
                C
0041                DO 40  I=1,20
```

```
FORTRAN IV G1   RELEASE 2.0              MAIN                DATE = 79206          22/16/33

0042              WRITE(6,901)
0043              WRITE(6,902)
0044              DO 40  J=1,54
0045              IPT=IPT+1
0046              WRITE(6,905) PMVALS(IPT),(POSITS(IPT,JPT),JPT=1,3)
0047           40 CONTINUE
            C
0048              STOP
0049              END
```

MASSES(I)	X	Y	Z
6.6700000000D+05	2.9981705034D+06	5.2813109998D+06	-1.6633286355D+06
6.6700000000D+05	2.9210425461D+06	5.3243574906D+06	-1.6633286355D+06
6.6700000000D+05	2.8432966826D+06	5.3662776874D+06	-1.6633286355D+06
6.6700000000D+05	2.7649453592D+06	5.4070627225D+06	-1.6633286355D+06
6.6700000000D+05	2.6860171490D+06	5.4467039684D+06	-1.6633286355D+06
6.6700000000D+05	2.6065167490D+06	5.4851930395D+06	-1.6633286355D+06
-6.6700000000D+05	2.5264649766D+06	5.5225217940D+06	-1.6633286355D+06
0.0	2.4458767655D+06	5.5586823356D+06	-1.6633286355D+06
-6.6700000000D+05	2.3647751627D+06	5.5936670149D+06	-1.6633286355D+06
-6.6700000000D+05	2.2831713244D+06	5.6274684315D+06	-1.6633286355D+06
6.6700000000D+05	2.2010845129D+06	5.6600794350D+06	-1.6633286355D+06
6.6700000000D+05	2.1185320924D+06	5.6914931272D+06	-1.6633286355D+06
-6.6700000000D+05	2.0355315258D+06	5.7217028629D+06	-1.6633286355D+06
-6.6700000000D+05	1.9521003708D+06	5.7507022516D+06	-1.6633286355D+06
-6.6700000000D+05	1.8682562759D+06	5.7784851589D+06	-1.6633286355D+06
6.6700000000D+05	1.7840169773D+06	5.8050457077D+06	-1.6633286355D+06
6.6700000000D+05	1.6994002946D+06	5.8303782795D+06	-1.6633286355D+06
-6.6700000000D+05	1.6144241274D+06	5.8544775156D+06	-1.6633286355D+06
-6.6700000000D+05	1.5291664510D+06	5.8773383181D+06	-1.6633286355D+06
6.6700000000D+05	1.4435465313D+06	5.8989558311D+06	-1.6633286355D+06
6.6700000000D+05	1.3575158307D+06	5.9193255417D+06	-1.6633286355D+06
6.6700000000D+05	1.2712651837D+06	5.9384430809D+06	-1.6633286355D+06
0.0	1.1847820140D+06	5.9563044248D+06	-1.6633286355D+06
-6.6700000000D+05	1.0980294199D+06	5.9729057951D+06	-1.6633286355D+06
-6.6700000000D+05	1.0110439528D+06	5.9882436798D+06	-1.6633286355D+06
-6.6700000000D+05	9.2384461348D+05	6.0023148340D+06	-1.6633286355D+06
6.6700000000D+05	8.3644984759D+05	6.0151162829D+06	-1.6633286355D+06
-6.6700000000D+05	7.4887814232D+05	6.0266453166D+06	-1.6633286355D+06
-6.6700000000D+05	6.6114862226D+05	6.0368994970D+06	-1.6633286355D+06
-6.6700000000D+05	5.7327804549D+05	6.0458760549D+06	-1.6633286355D+06
-6.6700000000D+05	4.8526679969D+05	6.0535748914D+06	-1.6633286355D+06
-6.6700000000D+05	3.9719289820D+05	6.0599925780D+06	-1.6633286355D+06
-6.6700000000D+05	3.0901497605D+05	6.0651283571D+06	-1.6633286355D+06
6.6700000000D+05	2.2077168606D+05	6.0689811424D+06	-1.6633286355D+06
-6.6700000000D+05	1.3248169488D+05	6.0715501187D+06	-1.6633286355D+06
6.6700000000D+05	4.4163679031D+04	6.0728347428D+06	-1.6633286355D+06
6.6700000000D+05	2.9559062221D+06	5.2597073305D+06	-1.7509242636D+06
-6.6700000000D+05	2.9090937635D+06	5.3025777359D+06	-1.7509242636D+06
-6.6700000000D+05	2.8316659263D+06	5.3443264544D+06	-1.7509242636D+06
-6.6700000000D+05	2.7538390895D+06	5.3849446547D+06	-1.7509242636D+06
6.6700000000D+05	2.6750297585D+06	5.4244237446D+06	-1.7509242636D+06
-6.6700000000D+05	2.5958545620D+06	5.4627553728D+06	-1.7509242636D+06
-6.6700000000D+05	2.5161302484D+06	5.4999314307D+06	-1.7509242636D+06
6.6700000000D+05	2.4358736824D+06	5.5359440544D+06	-1.7509242636D+06
-6.6700000000D+05	2.3551018410D+06	5.5707856258D+06	-1.7509242636D+06
-6.6700000000D+05	2.2738318104D+06	5.6044487747D+06	-1.7509242636D+06
6.6700000000D+05	2.1920807822D+06	5.6369263800D+06	-1.7509242636D+06
-6.6700000000D+05	2.1098660497D+06	5.6682115717D+06	-1.7509242636D+06
0.0	2.0272050043D+06	5.6982977318D+06	-1.7509242636D+06
0.0	1.9441151317D+06	5.7271784960D+06	-1.7509242636D+06
-6.6700000000D+05	1.8606140085D+06	5.7548477548D+06	-1.7509242636D+06
-6.6700000000D+05	1.7767192982D+06	5.7812995554D+06	-1.7509242636D+06
-6.6700000000D+05	1.6924487476D+06	5.8065285021D+06	-1.7509242636D+06
-6.6700000000D+05	1.6078201828D+06	5.8305292581D+06	-1.7509242636D+06

MASSES(I)	X	Y	Z
6.6700000000D+05	1.5228515060D+06	5.8532965464D+06	-1.7509242636D+06
-6.6700000000D+05	1.4375669100D+06	5.8748256510D+06	-1.7509242636D+06
-6.6700000000D+05	1.3519657600D+06	5.8951120176D+06	-1.7509242636D+06
-6.6700000000D+05	1.2660843794D+06	5.9141513549D+06	-1.7509242636D+06
-6.6700000000D+05	1.1799361560D+06	5.9319396354D+06	-1.7509242636D+06
-6.6700000000D+05	1.0935378335D+06	5.9484730962D+06	-1.7509242636D+06
-6.6700000000D+05	1.0069031682D+06	5.9637482400D+06	-1.7509242636D+06
6.6700000000D+05	9.2006554547D+05	5.9777618355D+06	-1.7509242636D+06
-6.6700000000D+05	8.3302827559D+05	5.9905109183D+06	-1.7509242636D+06
-6.6700000000D+05	7.4581479012D+05	6.0019927915D+06	-1.7509242636D+06
6.6700000000D+05	6.5844353786D+05	6.0122050262D+06	-1.7509242636D+06
-6.6700000000D+05	5.7093300160D+05	6.0211454623D+06	-1.7509242636D+06
-6.6700000000D+05	4.8330169116D+05	6.0288122085D+06	-1.7509242636D+06
-6.6700000000D+05	3.9556814556D+05	6.0352036430D+06	-1.7509242636D+06
-6.6700000000D+05	3.0775092299D+05	6.0403184137D+06	-1.7509242636D+06
-6.6700000000D+05	2.1986659997D+05	6.0441554388D+06	-1.7509242636D+06
6.6700000000D+05	1.3193976680D+05	6.0467159066D+06	-1.7509242636D+06
-6.6700000000D+05	4.3983023600D+04	6.0479932757D+06	-1.7509242636D+06
-6.6700000000D+05	2.9730132619D+06	5.2369962364D+06	-1.8381562613D+06
-6.6700000000D+05	2.8965224748D+06	5.2796815300D+06	-1.8381562613D+06
-6.6700000000D+05	2.8194239063D+06	5.3212499801D+06	-1.8381562613D+06
6.6700000000D+05	2.7417490446D+06	5.3616927935D+06	-1.8381562613D+06
-6.6700000000D+05	2.6634791438D+06	5.4010014151D+06	-1.8381562613D+06
6.6700000000D+05	2.5846458210D+06	5.4391675296D+06	-1.8381562613D+06
-6.6700000000D+05	2.5052657521D+06	5.4761830637D+06	-1.8381562613D+06
-6.6700000000D+05	2.4253557290D+06	5.5120401870D+06	-1.8381562613D+06
-6.6700000000D+05	2.3449326555D+06	5.5467313146D+06	-1.8381562613D+06
-6.6700000000D+05	2.2640135439D+06	5.5802491081D+06	-1.8381562613D+06
6.6700000000D+05	2.1826155117D+06	5.6125804772D+06	-1.8381562613D+06
6.6700000000D+05	2.1007557773D+06	5.6437365813D+06	-1.8381562613D+06
-6.6700000000D+05	2.0184516572D+06	5.6736928312D+06	-1.8381562613D+06
-6.6700000000D+05	1.9357205616D+06	5.7024488900D+06	-1.8381562613D+06
6.6700000000D+05	1.8525799912D+06	5.7299986747D+06	-1.8381562613D+06
-6.6700000000D+05	1.7690475331D+06	5.7563363575D+06	-1.8381562613D+06
6.6700000000D+05	1.6851408575D+06	5.7814563672D+06	-1.8381562613D+06
-6.6700000000D+05	1.6008777137D+06	5.8053533898D+06	-1.8381562613D+06
6.6700000000D+05	1.5162759205D+06	5.8280222704D+06	-1.8381562613D+06
-6.6700000000D+05	1.4313533920D+06	5.8494585136D+06	-1.8381562613D+06
-6.6700000000D+05	1.3461230746D+06	5.8696572849D+06	-1.8381562613D+06
-6.6700000000D+05	1.2606180024D+06	5.8886144115D+06	-1.8381562613D+06
-6.6700000000D+05	1.1748412640D+06	5.9063258833D+06	-1.8381562613D+06
-6.6700000000D+05	1.0888160041D+06	5.9227879537D+06	-1.8381562613D+06
6.6700000000D+05	1.0025604202D+06	5.9379971404D+06	-1.8381562613D+06
0.0	9.1609275880D+05	5.9519502260D+06	-1.8381562613D+06
-6.6700000000D+05	8.2943131052D+05	5.9646442590D+06	-1.8381562613D+06
-6.6700000000D+05	7.4259440753D+05	5.9760765542D+06	-1.8381562613D+06
-6.6700000000D+05	6.5560041899D+05	5.9862446931D+06	-1.8381562613D+06
6.6700000000D+05	5.6846774726D+05	5.9951465250D+06	-1.8381562613D+06
6.6700000000D+05	4.8121482400D+05	6.0027801666D+06	-1.8381562613D+06
6.6700000000D+05	3.9386010653D+05	6.0091440033D+06	-1.8381562613D+06
6.6700000000D+05	3.0642207335D+05	6.0142366880D+06	-1.8381562613D+06
6.6700000000D+05	2.1891922082D+05	6.0180571458D+06	-1.8381562613D+06
6.6700000000D+05	1.3137005897D+05	6.0206045662D+06	-1.8381562613D+06
6.6700000000D+05	4.3795107598D+04	6.0218734112D+06	-1.8381562613D+06

MASSES(I)	X	Y	Z
6.6700000000D+05	2.9594942018D+06	5.2131822603D+06	-1.9250064288D+06
-6.6700000000D+05	2.8833611926D+06	5.2556734528D+06	-1.9250064288D+06
6.6700000000D+05	2.8066182482D+06	5.2970528804D+06	-1.9250064288D+06
6.6700000000D+05	2.7292810027D+06	5.3373117899D+06	-1.9250064288D+06
6.6700000000D+05	2.6513076154D+06	5.3764416650D+06	-1.9250064288D+06
-6.6700000000D+05	2.5728427681D+06	5.4144342284D+06	-1.9250064288D+06
6.6700000000D+05	2.4938736609D+06	5.4512814432D+06	-1.9250064288D+06
6.6700000000D+05	2.4143270093D+06	5.4869755149D+06	-1.9250064288D+06
6.6700000000D+05	2.3342696403D+06	5.5215088930D+06	-1.9250064288D+06
6.6700000000D+05	2.2537184889D+06	5.5548742724D+06	-1.9250064288D+06
-6.6700000000D+05	2.1726905945D+06	5.5870645951D+06	-1.9250064288D+06
-6.6700000000D+05	2.0912030976D+06	5.6180730517D+06	-1.9250064288D+06
-6.6700000000D+05	2.0092752356D+06	5.6478930827D+06	-1.9250064288D+06
6.6700000000D+05	1.9269153397D+06	5.6765183802D+06	-1.9250064288D+06
6.6700000000D+05	1.8441558309D+06	5.7039428890D+06	-1.9250064288D+06
6.6700000000D+05	1.7610032165D+06	5.7301608076D+06	-1.9250064288D+06
-6.6700000000D+05	1.6774760863D+06	5.7551665900D+06	-1.9250064288D+06
6.6700000000D+05	1.5935591088D+06	5.7789549468D+06	-1.9250064288D+06
-6.6700000000D+05	1.5093816277D+06	5.8015208456D+06	-1.9250064288D+06
0.0	1.4248446580D+06	5.8228595131D+06	-1.9250064288D+06
6.6700000000D+05	1.340[illegible]8821D+06	5.8429664354D+06	-1.9250064288D+06
0.0	1.2548656404D+06	5.8618573590D+06	-1.9250064288D+06
6.6700000000D+05	1.1694989509D+06	5.8794662922D+06	-1.9250064288D+06
6.6700000000D+05	1.0838648762D+06	5.8958555053D+06	-1.9250064288D+06
-6.6700000000D+05	9.9800151886D+05	5.9109955318D+06	-1.9250064288D+06
-6.6700000000D+05	9.1192704813D+05	5.9248851692D+06	-1.9250064288D+06
6.6700000000D+05	8.2565967186D+05	5.9375214792D+06	-1.9250064288D+06
-6.6700000000D+05	7.3921763873D+05	5.9489017887D+06	-1.9250064288D+06
-6.6700000000D+05	6.5261923435D+05	5.9590236905D+06	-1.9250064288D+06
-6.6700000000D+05	5.6588217742D+05	5.9678850434D+06	-1.9250064288D+06
-6.6700000000D+05	4.7902661582D+05	5.9754839729D+06	-1.9250064288D+06
-6.6700000000D+05	3.9206912278D+05	5.9818188716D+06	-1.9250064288D+06
6.6700000000D+05	3.0502869294D+05	5.9868883593D+06	-1.9250064288D+06
-6.6700000000D+05	2.1792373851D+05	5.9906914837D+06	-1.9250064288D+06
-6.6700000000D+05	1.3077208534D+05	5.9932273204D+06	-1.9250064288D+06
-6.6700000000D+05	4.3593969013D+04	5.9944953728D+06	-1.9250064288D+06
-6.6700000000D+05	2.9455517479D+06	5.1882701689D+06	-2.0114566337D+06
-6.6700000000D+05	2.8695825534D+06	5.2305583099D+06	-2.0114566337D+06
-6.6700000000D+05	2.7932063384D+06	5.2717399988D+06	-2.0114566337D+06
6.6700000000D+05	2.7162392594D+06	5.3118065242D+06	-2.0114566337D+06
6.6700000000D+05	2.6386975976D+06	5.3507494104D+06	-2.0114566337D+06
6.6700000000D+05	2.5605977558D+06	5.3885604197D+06	-2.0114566337D+06
6.6700000000D+05	2.4819562551D+06	5.4252315538D+06	-2.0114566337D+06
-6.6700000000D+05	2.4027897309D+06	5.4607550552D+06	-2.0114566337D+06
6.6700000000D+05	2.3231149299D+06	5.4951234098D+06	-2.0114566337D+06
6.6700000000D+05	2.2429443706D+06	5.5283293468D+06	-2.0114566337D+06
-6.6700000000D+05	2.1623030175D+06	5.5603658425D+06	-2.0114566337D+06
6.6700000000D+05	2.0812099226D+06	5.5912261198D+06	-2.0114566337D+06
-6.6700000000D+05	1.9996715766D+06	5.6209036507D+06	-2.0114566337D+06
-6.6700000000D+05	1.9177102278D+06	5.6493921573D+06	-2.0114566337D+06
6.6700000000D+05	1.8353432139D+06	5.6766856132D+06	-2.0114566337D+06
-6.6700000000D+05	1.7525679587D+06	5.7027782450D+06	-2.0114566337D+06
6.6700000000D+05	1.6694619677D+06	5.7276645330D+06	-2.0114566337D+06
6.6700000000D+05	1.5859828252D+06	5.7513392130D+06	-2.0114566337D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	1.5021681899D+06	5.7737972768D+06	-2.0114566337D+06
-6.6700000000D+05	1.4180357918D+06	5.7950339738D+06	-2.0114566337D+06
-6.6700000000D+05	1.3336034279D+06	5.8150448116D+06	-2.0114566337D+06
6.6700000000D+05	1.2468889587D+06	5.8338255573D+06	-2.0114566337D+06
6.6700000000D+05	1.1639103043D+06	5.8513722380D+06	-2.0114566337D+06
6.6700000000D+05	1.0756854408D+06	5.8676811420D+06	-2.0114566337D+06
6.6700000000D+05	9.9323239632D+05	5.8827488193D+06	-2.0114566337D+06
6.6700000000D+05	9.0756924731D+05	5.8965720826D+06	-2.0114566337D+06
6.6700000000D+05	8.2171411459D+05	5.9091480078D+06	-2.0114566337D+06
6.6700000000D+05	7.3568515964D+05	5.9204739346D+06	-2.0114566337D+06
0.0	6.4950655069D+05	5.9305474671D+06	-2.0114566337D+06
6.6700000000D+05	5.6317860888D+05	5.9393664745D+06	-2.0114566337D+06
-6.6700000000D+05	4.7673750444D+05	5.9469290912D+06	-2.0114566337D+06
-6.6700000000D+05	3.9019555279D+05	5.9532337175D+06	-2.0114566337D+06
-6.6700000000D+05	3.0357106068D+05	5.9582790196D+06	-2.0114566337D+06
-6.6700000000D+05	2.1688235231D+05	5.9620639304D+06	-2.0114566337D+06
6.6700000000D+05	1.3014776549D+05	5.9645876491D+06	-2.0114566337D+06
6.6700000000D+05	4.3385647707D+04	5.9658496419D+06	-2.0114566337D+06
-6.6700000000D+05	2.9305887331D+06	5.1622649526D+06	-2.0974888147D+06
6.6700000000D+05	2.8551993172D+06	5.2043411324D+06	-2.0974888147D+06
-6.6700000000D+05	2.7792059233D+06	5.2453164059D+06	-2.0974888147D+06
-6.6700000000D+05	2.7026246273D+06	5.2851821054D+06	-2.0974888147D+06
-6.6700000000D+05	2.6254710283D+06	5.3239297976D+06	-2.0974888147D+06
-6.6700000000D+05	2.5477032471D+06	5.3615512867D+06	-2.0974888147D+06
0.0	2.4695159220D+06	5.3980386130D+06	-2.0974888147D+06
6.6700000000D+05	2.3907462049D+06	5.4333840603D+06	-2.0974888147D+06
6.6700000000D+05	2.3114707586D+06	5.4675601499D+06	-2.0974888147D+06
-6.6700000000D+05	2.2317063527D+06	5.5006196480D+06	-2.0974888147D+06
-6.6700000000D+05	2.1514698602D+06	5.5324955674D+06	-2.0974888147D+06
6.6700000000D+05	2.0707782541D+06	5.5632011635D+06	-2.0974888147D+06
6.6700000000D+05	1.9896486036D+06	5.5927299414D+06	-2.0974888147D+06
6.6700000000D+05	1.9080980704D+06	5.6210756548D+06	-2.0974888147D+06
6.6700000000D+05	1.8261439055D+06	5.6482323075D+06	-2.0974888147D+06
6.6700000000D+05	1.7438034452D+06	5.6741941549D+06	-2.0974888147D+06
6.6700000000D+05	1.6610941074D+06	5.6989557051D+06	-2.0974888147D+06
6.6700000000D+05	1.5780333882D+06	5.7225117203D+06	-2.0974888147D+06
6.6700000000D+05	1.4946338579D+06	5.7448572173D+06	-2.0974888147D+06
6.6700000000D+05	1.4109281574D+06	5.7659874694D+06	-2.0974888147D+06
6.6700000000D+05	1.3269189947D+06	5.7858980068D+06	-2.0974888147D+06
0.0	1.2420291408D+06	5.8045846175D+06	-2.0974888147D+06
6.6700000000D+05	1.1580764255D+06	5.8220433488D+06	-2.0974888147D+06
6.6700000000D+05	1.0732787354D+06	5.8382705075D+06	-2.0974888147D+06
6.6700000000D+05	9.8825400860D+05	5.8532626010D+06	-2.0974888147D+06
6.6700000000D+05	9.0302022926D+05	5.8670166376D+06	-2.0974888147D+06
6.6700000000D+05	8.1755429050D+05	5.8795295286D+06	-2.0974888147D+06
6.6700000000D+05	7.3199767778D+05	5.8907986663D+06	-2.0974888147D+06
6.6700000000D+05	6.4624508269D+05	5.9008217272D+06	-2.0974888147D+06
6.6700000000D+05	5.6035578334D+05	5.9095965310D+06	-2.0974888147D+06
6.6700000000D+05	4.7434794848D+05	5.9171212415D+06	-2.0974888147D+06
6.6700000000D+05	3.8823977188D+05	5.9233942670D+06	-2.0974888147D+06
6.6700000000D+05	3.0204946854D+05	5.9284142805D+06	-2.0974888147D+06
6.6700000000D+05	2.1579527082D+05	5.9321802201D+06	-2.0974888147D+06
6.6700000000D+05	1.2945542459D+05	5.9346912892D+06	-2.0974888147D+06
6.6700000000D+05	4.3168185406D+04	5.9359469565D+06	-2.0974888147D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	2.9152081168D+06	5.1351718244D+06	-2.1830849847D+06
6.6700000000D+05	2.8402143674D+06	5.1770271756D+06	-2.1830849847D+06
-6.6700000000D+05	2.7646198100D+06	5.2177873985D+06	-2.1830849847D+06
6.6700000000D+05	2.6884404356D+06	5.2574438707D+06	-2.1830849847D+06
0.0	2.6116923589D+06	5.2959882035D+06	-2.1830849847D+06
6.6700000000D+05	2.5343918146D+06	5.3334122435D+06	-2.1830849847D+06
-6.6700000000D+05	2.4565551553D+06	5.3697080739D+06	-2.1830849847D+06
6.6700000000D+05	2.3781968456D+06	5.4048660170D+06	-2.1830849847D+06
-6.6700000000D+05	2.2993394608D+06	5.4388846351D+06	-2.1830849847D+06
-6.6700000000D+05	2.2199936826D+06	5.4717507325D+06	-2.1830849847D+06
-6.6700000000D+05	2.1401782955D+06	5.5034593569D+06	-2.1830849847D+06
6.6700000000D+05	2.0599101834D+06	5.5340038067D+06	-2.1830849847D+06
6.6700000000D+05	1.9792063258D+06	5.5633776027D+06	-2.1830849847D+06
-6.6700000000D+05	1.8980837945D+06	5.5915745492D+06	-2.1830849847D+06
6.6700000000D+05	1.8165597499D+06	5.6185886756D+06	-2.1830849847D+06
0.0	1.7346514372D+06	5.6444142673D+06	-2.1830849847D+06
-6.6700000000D+05	1.6523761830D+06	5.6690458615D+06	-2.1830849847D+06
-6.6700000000D+05	1.5697513916D+06	5.6924782475D+06	-2.1830849847D+06
6.6700000000D+05	1.4867945410D+06	5.7147064086D+06	-2.1830849847D+06
6.6700000000D+05	1.4035231796D+06	5.7357258228D+06	-2.1830849847D+06
6.6700000000D+05	1.3199549224D+06	5.7555318636D+06	-2.1830849847D+06
6.6700000000D+05	1.2361074469D+06	5.7741204014D+06	-2.1830849847D+06
-6.6700000000D+05	1.1519984901D+06	5.7914875040D+06	-2.1830849847D+06
-6.6700000000D+05	1.0676458439D+06	5.8076294976D+06	-2.1830849847D+06
6.6700000000D+05	9.8306735204D+05	5.8225429677D+06	-2.1830849847D+06
6.6700000000D+05	8.9826090591D+05	5.8362247596D+06	-2.1830849847D+06
-6.6700000000D+05	8.1330444089D+05	5.8486719789D+06	-2.1830849847D+06
-6.6700000000D+05	7.2815593259D+05	5.8598819926D+06	-2.1830849847D+06
-6.6700000000D+05	6.4265339299D+05	5.8698524296D+06	-2.1830849847D+06
-6.6700000000D+05	5.5741486666D+05	5.8785811806D+06	-2.1830849847D+06
6.6700000000D+05	4.7185842694D+05	5.8860663991D+06	-2.1830849847D+06
-6.6700000000D+05	3.8620217210D+05	5.8923065019D+06	-2.1830849847D+06
0.0	3.0046422155D+05	5.8973001688D+06	-2.1830849847D+06
6.6700000000D+05	2.1466271195D+05	5.9010463437D+06	-2.1830849847D+06
6.6700000000D+05	1.2881579343D+05	5.9035442339D+06	-2.1830849847D+06
6.6700000000D+05	4.2941625712D+04	5.9047933111D+06	-2.1830849847D+06
-6.6700000000D+05	2.5992129647D+06	5.1069962197D+06	-2.2682272345D+06
-6.6700000000D+05	2.8246307102D+06	5.1486219195D+06	-2.2682272345D+06
6.6700000000D+05	2.7494509243D+06	5.1891584996D+06	-2.2682272345D+06
-6.6700000000D+05	2.6736695300D+06	5.2285973851D+06	-2.2682272345D+06
-6.6700000000D+05	2.5973625538D+06	5.2669302333D+06	-2.2682272345D+06
-6.6700000000D+05	2.5204861415D+06	5.3041489354D+06	-2.2682272345D+06
0.0	2.4430705553D+06	5.3402456183D+06	-2.2682272345D+06
6.6700000000D+05	2.3651501701D+06	5.3752126462D+06	-2.2682272345D+06
6.6700000000D+05	2.2867234701D+06	5.4090426223D+06	-2.2682272345D+06
-6.6700000000D+05	2.2078130455D+06	5.4417283904D+06	-2.2682272345D+06
-6.6700000000D+05	2.1284355886D+06	5.4732630362D+06	-2.2682272345D+06
-6.6700000000D+05	2.0486078907D+06	5.5036398891D+06	-2.2682272345D+06
6.6700000000D+05	1.9683466381D+06	5.5328525232D+06	-2.2682272345D+06
-6.6700000000D+05	1.8876694090D+06	5.5608947593D+06	-2.2682272345D+06
6.6700000000D+05	1.8065926697D+06	5.5877606640D+06	-2.2682272345D+06
-6.6700000000D+05	1.7251337766D+06	5.6134445568D+06	-2.2682272345D+06
6.6700000000D+05	1.6433094434D+06	5.6379410025D+06	-2.2682272345D+06
-6.6700000000D+05	1.5611384962D+06	5.6612448200D+06	-2.2682272345D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	1.4786368130D+06	5.6833510796D+06	-2.2682272345D+06
6.6700000000D+05	1.3958223440D+06	5.7042551050D+06	-2.2682272345D+06
6.6700000000D+05	1.3127126081D+06	5.7239524742D+06	-2.2682272345D+06
-6.6700000000D+05	1.2293251861D+06	5.7424390206D+06	-2.2682272345D+06
6.6700000000D+05	1.1456777173D+06	5.7597108336D+06	-2.2682272345D+06
-6.6700000000D+05	1.0617878963D+06	5.7757642596D+06	-2.2682272345D+06
-6.6700000000D+05	9.7767346883D+05	5.7905959026D+06	-2.2682272345D+06
-6.6700000000D+05	8.9335222805D+05	5.8042026253D+06	-2.2682272345D+06
6.6700000000D+05	8.0584201097D+05	5.8165815493D+06	-2.2682272345D+06
-6.6700000000D+05	7.2416069457D+05	5.8277300561D+06	-2.2682272345D+06
-6.6700000000D+05	6.3932619202D+05	5.8376457874D+06	-2.2682272345D+06
-6.6700000000D+05	5.5435644886D+05	5.8463206455D+06	-2.2682272345D+06
6.6700000000D+05	4.6926943928D+05	5.8537707943D+06	-2.2682272345D+06
6.6700000000D+05	3.8408316224D+05	5.8599766589D+06	-2.2682272345D+06
0.0	2.9881563774D+05	5.8649429267D+06	-2.2682272345D+06
-6.6700000000D+05	2.1348490293D+05	5.8686685470D+06	-2.2682272345D+06
-6.6700000000D+05	1.2810900834D+05	5.8711527319D+06	-2.2682272345D+06
-6.6700000000D+05	4.2706614069D+04	5.8723949556D+06	-2.2682272345D+06
-6.6700000000D+05	2.8826605484D+06	5.0777437957D+06	-2.3528977357D+06
6.6700000000D+05	2.8064514746D+06	5.1191310671D+06	-2.3528977357D+06
6.6700000000D+05	2.7337023118D+06	5.1594354573D+06	-2.3528977357D+06
-6.6700000000D+05	2.6583748722D+06	5.1986484404D+06	-2.3528977357D+06
6.6700000000D+05	2.5824853901D+06	5.2367017214D+06	-2.3528977357D+06
-6.6700000000D+05	2.5060490192D+06	5.2737672361D+06	-2.3528977357D+06
-6.6700000000D+05	2.4290828282D+06	5.3096571623D+06	-2.3528977357D+06
6.6700000000D+05	2.3516027985D+06	5.3444239022D+06	-2.3528977357D+06
-6.6700000000D+05	2.2736253198D+06	5.3780601032D+06	-2.3528977357D+06
-6.6700000000D+05	2.1951668871D+06	5.4105586501D+06	-2.3528977357D+06
-6.6700000000D+05	2.1162440973D+06	5.4419126682D+06	-2.3528977357D+06
-6.6700000000D+05	2.0368730453D+06	5.4721155252D+06	-2.3528977357D+06
0.0	1.9570723210D+06	5.5011608319D+06	-2.3528977357D+06
6.6700000000D+05	1.8768570051D+06	5.5290424442D+06	-2.3528977357D+06
6.6700000000D+05	1.7962446662D+06	5.5557544643D+06	-2.3528977357D+06
-6.6700000000D+05	1.7152523565D+06	5.5812912414D+06	-2.3528977357D+06
-6.6700000000D+05	1.6338972090D+06	5.6056473737D+06	-2.3528977357D+06
6.6700000000D+05	1.5521964332D+06	5.6288177090D+06	-2.3528977357D+06
6.6700000000D+05	1.4701673117D+06	5.6507973458D+06	-2.3528977357D+06
-6.6700000000D+05	1.3878271967D+06	5.6715816348D+06	-2.3528977357D+06
-6.6700000000D+05	1.3051935061D+06	5.6911601792D+06	-2.3528977357D+06
-6.6700000000D+05	1.2222637196D+06	5.7095468363D+06	-2.3528977357D+06
6.6700000000D+05	1.1391153764D+06	5.7267197179D+06	-2.3528977357D+06
6.6700000000D+05	1.0557060689D+06	5.7426811912D+06	-2.3528977357D+06
6.6700000000D+05	9.7207344142D+05	5.7574278799D+06	-2.3528977357D+06
-6.6700000000D+05	8.8823516527D+05	5.7709560645D+06	-2.3528977357D+06
-6.6700000000D+05	8.0420903526D+05	5.7832646832D+06	-2.3528977357D+06
6.6700000000D+05	7.2001276595D+05	5.7943493323D+06	-2.3528977357D+06
6.6700000000D+05	6.3506418797D+05	5.8042082671D+06	-2.3528977357D+06
-6.6700000000D+05	5.5118114401D+05	5.8128394021D+06	-2.3528977357D+06
6.6700000000D+05	4.6658150531D+05	5.8202409114D+06	-2.3528977357D+06
-6.6700000000D+05	3.8188321077D+05	5.8264112294D+06	-2.3528977357D+06
6.6700000000D+05	2.9716404812D+05	5.8313490568D+06	-2.3528977357D+06
6.6700000000D+05	2.1226200023D+05	5.8350533311D+06	-2.3528977357D+06
-6.6700000000D+05	1.2737521124D+05	5.8375232866D+06	-2.3528977357D+06
-6.6700000000D+05	4.2481397786D+04	5.8387583452D+06	-2.3528977357D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	2.8653921450D+06	5.0474204307D+06	-2.4370787448D+06
-6.6700000000D+05	2.7916799118D+06	5.0865605449D+06	-2.4370787448D+06
-6.6700000000D+05	2.7173771374D+06	5.1286242445D+06	-2.4370787448D+06
-6.6700000000D+05	2.6424995396D+06	5.1676030548D+06	-2.4370787448D+06
6.6700000000D+05	2.5670629576D+06	5.2054887302D+06	-2.4370787448D+06
-6.6700000000D+05	2.4910833490D+06	5.2422732567D+06	-2.4370787448D+06
-6.6700000000D+05	2.4145767862D+06	5.2779488528D+06	-2.4370787448D+06
6.6700000000D+05	2.3375594531D+06	5.3125079720D+06	-2.4370787448D+06
-6.6700000000D+05	2.2600476417D+06	5.3459433037D+06	-2.4370787448D+06
-6.6700000000D+05	2.1820577486D+06	5.3782477751D+06	-2.4370787448D+06
-6.6700000000D+05	2.1036062714D+06	5.4094145528D+06	-2.4370787448D+06
-6.6700000000D+05	2.0247098054D+06	5.4394370437D+06	-2.4370787448D+06
-6.6700000000D+05	1.9453850401D+06	5.4683088972D+06	-2.4370787448D+06
6.6700000000D+05	1.8656487555D+06	5.4960240050D+06	-2.4370787448D+06
-6.6700000000D+05	1.7855178188D+06	5.5225765063D+06	-2.4370787448D+06
-6.6700000000D+05	1.7050091805D+06	5.5479607824D+06	-2.4370787448D+06
6.6700000000D+05	1.6241398712D+06	5.5721714643D+06	-2.4370787448D+06
0.0	1.5429269976D+06	5.5952534305D+06	-2.4370787448D+06
6.6700000000D+05	1.4613877392D+06	5.6170518090D+06	-2.4370787448D+06
6.6700000000D+05	1.3795393444D+06	5.6377115779D+06	-2.4370787448D+06
-6.6700000000D+05	1.2973991272D+06	5.6571795670D+06	-2.4370787448D+06
-6.6700000000D+05	1.2149764631D+06	5.6754594582D+06	-2.4370787448D+06
6.6700000000D+05	1.1323127860D+06	5.6925207864D+06	-2.4370787448D+06
6.6700000000D+05	1.0494015837D+06	5.7083869407D+06	-2.4370787448D+06
6.6700000000D+05	9.6626839514D+05	5.7230455649D+06	-2.4370787448D+06
-6.6700000000D+05	8.8293080584D+05	5.7364935583D+06	-2.4370787448D+06
0.0	7.9940644470D+05	5.7487280754D+06	-2.4370787448D+06
6.6700000000D+05	7.1571293033D+05	5.7597405290D+06	-2.4370787448D+06
-6.6700000000D+05	6.3186811673D+05	5.7695465881D+06	-2.4370787448D+06
6.6700000000D+05	5.4788959018D+05	5.7781261795D+06	-2.4370787448D+06
6.6700000000D+05	4.6379516519D+05	5.7854834883D+06	-2.4370787448D+06
-6.6700000000D+05	3.7960263075D+05	5.7916169583D+06	-2.4370787448D+06
6.6700000000D+05	2.9532579663D+05	5.7965252919D+06	-2.4370787448D+06
6.6700000000D+05	2.1099448958D+05	5.8002074510D+06	-2.4370787448D+06
-6.6700000000D+05	1.2661454958D+05	5.8026620505D+06	-2.4370787448D+06
-6.6700000000D+05	4.2207826020D+04	5.8038903891D+06	-2.4370787448D+06
-6.6700000000D+05	2.8475732365D+06	5.0160322234D+06	-2.5207526061D+06
6.6700000000D+05	2.7743193949D+06	5.0569165011D+06	-2.5207526061D+06
6.6700000000D+05	2.7004786844D+06	5.0967310581D+06	-2.5207526061D+06
6.6700000000D+05	2.6260667252D+06	5.1354674723D+06	-2.5207526061D+06
6.6700000000D+05	2.5510992578D+06	5.1731175494D+06	-2.5207526061D+06
-6.6700000000D+05	2.4755921408D+06	5.2096733253D+06	-2.5207526061D+06
6.6700000000D+05	2.3995613465D+06	5.2451270069D+06	-2.5207526061D+06
6.6700000000D+05	2.3230229583D+06	5.2794712745D+06	-2.5207526061D+06
6.6700000000D+05	2.2459931668D+06	5.3126986532D+06	-2.5207526061D+06
6.6700000000D+05	2.1684882665D+06	5.3448022640D+06	-2.5207526061D+06
6.6700000000D+05	2.0905246526D+06	5.3757752260D+06	-2.5207526061D+06
0.0	2.0121168172D+06	5.4056110172D+06	-2.5207526061D+06
-6.6700000000D+05	1.9332873459D+06	5.4343033263D+06	-2.5207526061D+06
0.0	1.8540469144D+06	5.4618460838D+06	-2.5207526061D+06
6.6700000000D+05	1.7744142850D+06	5.4882334634D+06	-2.5207526061D+06
-6.6700000000D+05	1.6944063028D+06	5.5134598633D+06	-2.5207526061D+06
6.6700000000D+05	1.6140398925D+06	5.5375200071D+06	-2.5207526061D+06
-6.6700000000D+05	1.5333320544D+06	5.5604087453D+06	-2.5207526061D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	1.4522998611D+06	5.5821212560D+06	-2.5207526061D+06
0.0	1.3709604539D+06	5.6026529464D+06	-2.5207526061D+06
-6.6700000000D+05	1.2893310591D+06	5.6219994731D+06	-2.5207526061D+06
6.6700000000D+05	1.2074268841D+06	5.6401567437D+06	-2.5207526061D+06
6.6700000000D+05	1.1252713142D+06	5.6571120917D+06	-2.5207526061D+06
-6.6700000000D+05	1.0426757088D+06	5.6728864654D+06	-2.5207526061D+06
6.6700000000D+05	9.6025949750D+05	5.6874558725D+06	-2.5207526061D+06
0.0	8.7744015660D+05	5.7008202371D+06	-2.5207526061D+06
-6.6700000000D+05	7.9443520539D+05	5.7129786721D+06	-2.5207526061D+06
6.6700000000D+05	7.1126220241D+05	5.7239286057D+06	-2.5207526061D+06
-6.6700000000D+05	6.2793674177D+05	5.7330677215D+06	-2.5207526061D+06
-6.6700000000D+05	5.4448244939D+05	5.7421939593D+06	-2.5207526061D+06
-6.6700000000D+05	4.6091047930D+05	5.7495055155D+06	-2.5207526061D+06
-6.6700000000D+05	3.7724200986D+05	5.7556008435D+06	-2.5207526061D+06
6.6700000000D+05	2.9349324011D+05	5.7604786539D+06	-2.5207526061D+06
-6.6700000000D+05	2.0966238593D+05	5.7641379148D+06	-2.5207526061D+06
6.6700000000D+05	1.2582717635D+05	5.7665778522D+06	-2.5207526061D+06
-6.6700000000D+05	4.1545349769D+04	5.7677979499D+06	-2.5207526061D+06
6.6700000000D+05	2.8291534100D+06	4.9835854923D+06	-2.6039017553D+06
6.6700000000D+05	2.7563734160D+06	5.0242053057D+06	-2.6039017553D+06
6.6700000000D+05	2.6830103545D+06	5.0637623181D+06	-2.6039017553D+06
6.6700000000D+05	2.6096757368D+06	5.1022481617D+06	-2.6039017553D+06
0.0	2.5345972044D+06	5.1396540954D+06	-2.6039017553D+06
6.6700000000D+05	2.4595765131D+06	5.1759740063D+06	-2.6039017553D+06
6.6700000000D+05	2.3845595320D+06	5.2111984117D+06	-2.6039017553D+06
6.6700000000D+05	2.3079962404D+06	5.2453204602D+06	-2.6039017553D+06
6.6700000000D+05	2.2314647242D+06	5.2783329339D+06	-2.6039017553D+06
6.6700000000D+05	2.1544611725D+06	5.3102288494D+06	-2.6039017553D+06
6.6700000000D+05	2.0776015744D+06	5.3410014595D+06	-2.6039017553D+06
6.6700000000D+05	1.9991032153D+06	5.3706442547D+06	-2.6039017553D+06
0.0	1.9207816737D+06	5.3991509646D+06	-2.6039017553D+06
6.6700000000D+05	1.8420538172D+06	5.4265155589D+06	-2.6039017553D+06
0.0	1.7629362999D+06	5.4527322493D+06	-2.6039017553D+06
-6.6700000000D+05	1.6834458577D+06	5.4777954891D+06	-2.6039017553D+06
6.6700000000D+05	1.6035993600D+06	5.5016999775D+06	-2.6039017553D+06
6.6700000000D+05	1.5234135350D+06	5.5244406574D+06	-2.6039017553D+06
-6.6700000000D+05	1.4429055070D+06	5.5460127165D+06	-2.6039017553D+06
-6.6700000000D+05	1.3620922523D+06	5.5664115975D+06	-2.6039017553D+06
0.0	1.2809908058D+06	5.5856329792D+06	-2.6039017553D+06
6.6700000000D+05	1.1996135035D+06	5.6036727976D+06	-2.6039017553D+06
-6.6700000000D+05	1.1179923786D+06	5.6205272366D+06	-2.6039017553D+06
-6.6700000000D+05	1.0361297578D+06	5.6361927310D+06	-2.6039017553D+06
6.6700000000D+05	9.5404795811D+05	5.6506659669D+06	-2.6039017553D+06
6.6700000000D+05	8.7176434281D+05	5.6639438827D+06	-2.6039017553D+06
6.6700000000D+05	7.8929631784D+05	5.6760236697D+06	-2.6039017553D+06
6.6700000000D+05	7.0666132817D+05	5.6869027725D+06	-2.6039017553D+06
6.6700000000D+05	6.2387854069D+05	5.6965788898D+06	-2.6039017553D+06
6.6700000000D+05	5.4096040750D+05	5.7050499747D+06	-2.6039017553D+06
-6.6700000000D+05	4.5792952823D+05	5.7123142354D+06	-2.6039017553D+06
-6.6700000000D+05	3.7480178031D+05	5.7183701351D+06	-2.6039017553D+06
-6.6700000000D+05	2.9159474827D+05	5.7232163928D+06	-2.6039017553D+06
6.6700000000D+05	2.0832603340D+05	5.7268519834D+06	-2.6039017553D+06
6.6700000000D+05	1.2501325050D+05	5.7292761378D+06	-2.6039017553D+06
6.6700000000D+05	4.1674021869D+04	5.7304883433D+06	-2.6039017553D+06

MASSES(I)	X	Y	Z
6.6700000000D+05	2.8101363768D+06	4.9500867748D+06	-2.6865087231D+06
-6.6700000000D+05	2.7373455987D+06	4.9904335495D+06	-2.6865087231D+06
0.0	2.6649756672D+06	5.0297246671D+06	-2.6865087231D+06
6.6700000000D+05	2.5915419571D+06	5.0679518162D+06	-2.6865087231D+06
-6.6700000000D+05	2.5175661222D+06	5.1051069163D+06	-2.6865087231D+06
6.6700000000D+05	2.4430450923D+06	5.1411820897D+06	-2.6865087231D+06
6.6700000000D+05	2.3680144700D+06	5.1761697233D+06	-2.6865087231D+06
6.6700000000D+05	2.2924823269D+06	5.2100624099D+06	-2.6865087231D+06
6.6700000000D+05	2.2164652411D+06	5.2428529799D+06	-2.6865087231D+06
6.6700000000D+05	2.1399792927D+06	5.2745344971D+06	-2.6865087231D+06
-6.6700000000D+05	2.0630406613D+06	5.3051002596D+06	-2.6865087231D+06
6.6700000000D+05	1.9856656222D+06	5.3345438015D+06	-2.6865087231D+06
6.6700000000D+05	1.9078705432D+06	5.3628588947D+06	-2.6865087231D+06
-6.6700000000D+05	1.8296718805D+06	5.3900395494D+06	-2.6865087231D+06
-6.6700000000D+05	1.7510861761D+06	5.4160800158D+06	-2.6865087231D+06
-6.6700000000D+05	1.6721300536D+06	5.4409747856D+06	-2.6865087231D+06
6.6700000000D+05	1.5928202152D+06	5.4647185926D+06	-2.6865087231D+06
-6.6700000000D+05	1.5131734378D+06	5.4873064140D+06	-2.6865087231D+06
-6.6700000000D+05	1.4332065695D+06	5.5087334718D+06	-2.6865087231D+06
-6.6700000000D+05	1.3529385262D+06	5.5289952333D+06	-2.6865087231D+06
-6.6700000000D+05	1.2723882879D+06	5.5480874125D+06	-2.6865087231D+06
6.6700000000D+05	1.1915548952D+06	5.5660059707D+06	-2.6865087231D+06
6.6700000000D+05	1.1104774456D+06	5.5827471174D+06	-2.6865087231D+06
-6.6700000000D+05	1.0291650899D+06	5.5983073113D+06	-2.6865087231D+06
-6.6700000000D+05	9.4763502849D+05	5.6126832609D+06	-2.6865087231D+06
6.6700000000D+05	8.6590450806D+05	5.6258719250D+06	-2.6865087231D+06
6.6700000000D+05	7.8399081753D+05	5.6378705139D+06	-2.6865087231D+06
-6.6700000000D+05	7.0191128461D+05	5.6486764895D+06	-2.6865087231D+06
6.6700000000D+05	6.1968327207D+05	5.6582875657D+06	-2.6865087231D+06
-6.6700000000D+05	5.3732417413D+05	5.6667017096D+06	-2.6865087231D+06
-6.6700000000D+05	4.5485141270D+05	5.6739171413D+06	-2.6865087231D+06
-6.6700000000D+05	3.7228243375D+05	5.6799323344D+06	-2.6865087231D+06
6.6700000000D+05	2.8963470362D+05	5.6847460165D+06	-2.6865087231D+06
-6.6700000000D+05	2.0692570527D+05	5.6883571694D+06	-2.6865087231D+06
6.6700000000D+05	1.2417293466D+05	5.6907650291D+06	-2.6865087231D+06
6.6700000000D+05	4.1393069990D+04	5.6919690863D+06	-2.6865087231D+06
0.0	2.7905259720D+06	4.9155428266D+06	-2.7685561384D+06
6.6700000000D+05	2.7187396717D+06	4.9556080433D+06	-2.7685561384D+06
-6.6700000000D+05	2.6463782597D+06	4.9946249697D+06	-2.7685561384D+06
-6.6700000000D+05	2.5734570430D+06	5.0325853524D+06	-2.7685561384D+06
-6.6700000000D+05	2.4999144471D+06	5.0694811614D+06	-2.7685561384D+06
-6.6700000000D+05	2.4259970127D+06	5.1053045919D+06	-2.7685561384D+06
6.6700000000D+05	2.3514893922D+06	5.1400480659D+06	-2.7685561384D+06
-6.6700000000D+05	2.2764843467D+06	5.1737042340D+06	-2.7685561384D+06
-6.6700000000D+05	2.2009977424D+06	5.2062659706D+06	-2.7685561384D+06
-6.6700000000D+05	2.1250455476D+06	5.2377264058D+06	-2.7685561384D+06
6.6700000000D+05	2.0486438289D+06	5.2680788665D+06	-2.7685561384D+06
-6.6700000000D+05	1.9718057479D+06	5.2973169381D+06	-2.7685561384D+06
-6.6700000000D+05	1.8945565581D+06	5.3254344357D+06	-2.7685561384D+06
6.6700000000D+05	1.8169036012D+06	5.3524254115D+06	-2.7685561384D+06
-6.6700000000D+05	1.7388663034D+06	5.3782841558D+06	-2.7685561384D+06
-6.6700000000D+05	1.6604611726D+06	5.4030051986D+06	-2.7685561384D+06
6.6700000000D+05	1.5817647942D+06	5.4265833105D+06	-2.7685561384D+06
-6.6700000000D+05	1.5026138280D+06	5.4490135038D+06	-2.7685561384D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	1.4232050048D+06	5.4702910339D+06	-2.7685561384D+06
-6.6700000000D+05	1.3434951222D+06	5.4904113997D+06	-2.7685561384D+06
-6.6700000000D+05	1.2635010418D+06	5.5093703450D+06	-2.7685561384D+06
-6.6700000000D+05	1.1832396853D+06	5.5271638594D+06	-2.7685561384D+06
-6.6700000000D+05	1.1027280309D+06	5.5437881788D+06	-2.7685561384D+06
-6.6700000000D+05	1.0219831096D+06	5.5592397866D+06	-2.7685561384D+06
6.6700000000D+05	9.4102200194D+05	5.5735154143D+06	-2.7685561384D+06
-6.6700000000D+05	8.5986183411D+05	5.5866120420D+06	-2.7685561384D+06
-6.6700000000D+05	7.7851977442D+05	5.5985268993D+06	-2.7685561384D+06
-6.6700000000D+05	6.9701302966D+05	5.6092574658D+06	-2.7685561384D+06
6.6700000000D+05	6.1535884145D+05	5.6188014716D+06	-2.7685561384D+06
-6.6700000000D+05	5.3357448261D+05	5.6271568973D+06	-2.7685561384D+06
6.6700000000D+05	4.5167725347D+05	5.6343219769D+06	-2.7685561384D+06
6.6700000000D+05	3.6965447827D+05	5.6402951933D+06	-2.7685561384D+06
6.6700000000D+05	2.8761350144D+05	5.6450752834D+06	-2.7685561384D+06
-6.6700000000D+05	2.0548166396D+05	5.6486612360D+06	-2.7685561384D+06
6.6700000000D+05	1.2330639967D+05	5.6510522925D+06	-2.7685561384D+06
-6.6700000000D+05	4.1105031622D+04	5.6522479473D+06	-2.7685561384D+06
-6.6700000000D+05	2.7703261544D+06	4.8799606211D+06	-2.8500267322D+06
-6.6700000000D+05	2.6990594945D+06	4.9197358171D+06	-2.8500267322D+06
6.6700000000D+05	2.6272218860D+06	4.9584703111D+06	-2.8500267322D+06
-6.6700000000D+05	2.5548265251D+06	4.9961529055D+06	-2.8500267322D+06
-6.6700000000D+05	2.4818947256D+06	5.0327846403D+06	-2.8500267322D+06
0.0	2.4084359156D+06	5.0683487552D+06	-2.8500267322D+06
-6.6700000000D+05	2.3344676345D+06	5.1028407312D+06	-2.8500267322D+06
-6.6700000000D+05	2.2600655290D+06	5.1362532719D+06	-2.8500267322D+06
-6.6700000000D+05	2.1850653507D+06	5.1685793094D+06	-2.8500267322D+06
-6.6700000000D+05	2.1096629520D+06	5.1998120056D+06	-2.8500267322D+06
6.6700000000D+05	2.0338142834D+06	5.2299447535D+06	-2.8500267322D+06
6.6700000000D+05	1.9575355896D+06	5.2589711791D+06	-2.8500267322D+06
6.6700000000D+05	1.8808424063D+06	5.2868851423D+06	-2.8500267322D+06
0.0	1.8037515568D+06	5.3136807381D+06	-2.8500267322D+06
6.6700000000D+05	1.7262791486D+06	5.3393522983D+06	-2.8500267322D+06
-6.6700000000D+05	1.6484415701D+06	5.3638943926D+06	-2.8500267322D+06
6.6700000000D+05	1.5702552865D+06	5.3873018293D+06	-2.8500267322D+06
-6.6700000000D+05	1.4917368372D+06	5.4095696569D+06	-2.8500267322D+06
6.6700000000D+05	1.4129028317D+06	5.4306931650D+06	-2.8500267322D+06
6.6700000000D+05	1.3337699461D+06	5.4506678853D+06	-2.8500267322D+06
6.6700000000D+05	1.2543549200D+06	5.4694895922D+06	-2.8500267322D+06
6.6700000000D+05	1.1746745524D+06	5.4871543044D+06	-2.8500267322D+06
-6.6700000000D+05	1.0947456987D+06	5.5036582851D+06	-2.8500267322D+06
-6.6700000000D+05	1.0145852688D+06	5.5189980432D+06	-2.8500267322D+06
6.6700000000D+05	9.3421021339D+05	5.5331703337D+06	-2.8500267322D+06
6.6700000000D+05	8.5363754075D+05	5.5461721587D+06	-2.8500267322D+06
-6.6700000000D+05	7.7288429292D+05	5.5580007676D+06	-2.8500267322D+06
6.6700000000D+05	6.9196755212D+05	5.5686556588D+06	-2.8500267322D+06
-6.6700000000D+05	6.1090443518D+05	5.5781285783D+06	-2.8500267322D+06
6.6700000000D+05	5.2971208988D+05	5.5864255219D+06	-2.8500267322D+06
-6.6700000000D+05	4.4840769130D+05	5.5935367351D+06	-2.8500267322D+06
6.6700000000D+05	3.6700843829D+05	5.5994667131D+06	-2.8500267322D+06
6.6700000000D+05	2.8553154974D+05	5.6042122014D+06	-2.8500267322D+06
6.6700000000D+05	2.0399426094D+05	5.6077721963D+06	-2.8500267322D+06
6.6700000000D+05	1.2241381999D+05	5.6101459447D+06	-2.8500267322D+06
6.6700000000D+05	4.0807484081D+04	5.6113329445D+06	-2.8500267322D+06

MASSES(I)	X	Y	Z
6.6700000000D+05	2.7495410054D+06	4.8433473478D+06	-2.9309033404D+06
-6.6700000000D+05	2.6788090436D+06	4.8828241192D+06	-2.9309033404D+06
6.6700000000D+05	2.6075104169D+06	4.9212679969D+06	-2.9309033404D+06
6.6700000000D+05	2.5355602075D+06	4.9586708485D+06	-2.9309033404D+06
-6.6700000000D+05	2.4632736143D+06	4.9950247620D+06	-2.9309033404D+06
6.6700000000D+05	2.3903659497D+06	5.0303220472D+06	-2.9309033404D+06
6.6700000000D+05	2.3169526363D+06	5.0645552374D+06	-2.9309033404D+06
6.6700000000D+05	2.2430492036D+06	5.0977170912D+06	-2.9309033404D+06
6.6700000000D+05	2.1686712850D+06	5.1298005935D+06	-2.9309033404D+06
6.6700000000D+05	2.0938346140D+06	5.1607989576D+06	-2.9309033404D+06
-6.6700000000D+05	2.0185550213D+06	5.1907056261D+06	-2.9309033404D+06
-6.6700000000D+05	1.9428464313D+06	5.2195142729D+06	-2.9309033404D+06
6.6700000000D+05	1.8667308586D+06	5.2472188036D+06	-2.9309033404D+06
6.6700000000D+05	1.7902164046D+06	5.2738133580D+06	-2.9309033404D+06
6.6700000000D+05	1.7133272551D+06	5.2992923101D+06	-2.9309033404D+06
6.6700000000D+05	1.6360730748D+06	5.3236502704D+06	-2.9309033404D+06
6.6700000000D+05	1.5584740058D+06	5.3468820863D+06	-2.9309033404D+06
-6.6700000000D+05	1.4805446632D+06	5.3689828433D+06	-2.9309033404D+06
0.0	1.4023021319D+06	5.3899478664D+06	-2.9309033404D+06
6.6700000000D+05	1.3237629036D+06	5.4097727207D+06	-2.9309033404D+06
-6.6700000000D+05	1.2449437703D+06	5.4284552125D+06	-2.9309033404D+06
6.6700000000D+05	1.1658612271D+06	5.4459853902D+06	-2.9309033404D+06
0.0	1.0865320621D+06	5.4623555452D+06	-2.9309033404D+06
0.0	1.0069730583D+06	5.4775902124D+06	-2.9309033404D+06
-6.6700000000D+05	9.2720103924D+05	5.4916501713D+06	-2.9309033404D+06
-6.6700000000D+05	8.4723208568D+05	5.5045604465D+06	-2.9309033404D+06
-6.6700000000D+05	7.6706551174D+05	5.5163003081D+06	-2.9309033404D+06
-6.6700000000D+05	6.8677587148D+05	5.5268732728D+06	-2.9309033404D+06
-6.6700000000D+05	6.0632095331D+05	5.5362771041D+06	-2.9309033404D+06
6.6700000000D+05	5.2573777634D+05	5.5445096126D+06	-2.9309033404D+06
-6.6700000000D+05	4.4504338682D+05	5.5515696569D+06	-2.9309033404D+06
-6.6700000000D+05	3.6425435455D+05	5.5574551435D+06	-2.9309033404D+06
-6.6700000000D+05	2.8338926920D+05	5.5621650275D+06	-2.9309033404D+06
-6.6700000000D+05	2.0246373678D+05	5.5656983126D+06	-2.9309033404D+06
6.6700000000D+05	1.2149537597D+05	5.5680542513D+06	-2.9309033404D+06
6.6700000000D+05	4.0501314436D+04	5.5692323452D+06	-2.9309033404D+06
-6.6700000000D+05	2.7281747291D+06	4.8057104123D+06	-3.0111689081D+06
6.6700000000D+05	2.6579924150D+06	4.8446804156D+06	-3.0111689081D+06
6.6700000000D+05	2.5872478393D+06	4.8830255516D+06	-3.0111689081D+06
6.6700000000D+05	2.5159559673D+06	4.9201377512D+06	-3.0111689081D+06
6.6700000000D+05	2.4441318796D+06	4.9562091637D+06	-3.0111689081D+06
6.6700000000D+05	2.3717907696D+06	4.9912321590D+06	-3.0111689081D+06
6.6700000000D+05	2.2989479402D+06	5.0251993282D+06	-3.0111689081D+06
6.6700000000D+05	2.2256138002D+06	5.0581034861D+06	-3.0111689081D+06
-6.6700000000D+05	2.1516188614D+06	5.0899376722D+06	-3.0111689081D+06
6.6700000000D+05	2.0775637351D+06	5.1206951526D+06	-3.0111689081D+06
-6.6700000000D+05	2.0028691290D+06	5.1503694209D+06	-3.0111689081D+06
6.6700000000D+05	1.9277508436D+06	5.1785541999D+06	-3.0111689081D+06
6.6700000000D+05	1.8522247693D+06	5.2064434428D+06	-3.0111689081D+06
6.6700000000D+05	1.7763066825D+06	5.2326313348D+06	-3.0111689081D+06
-6.6700000000D+05	1.7000132420D+06	5.2581122938D+06	-3.0111689081D+06
-6.6700000000D+05	1.6233599884D+06	5.2822809723D+06	-3.0111689081D+06
-6.6700000000D+05	1.5403633350D+06	5.3053322569D+06	-3.0111689081D+06
6.6700000000D+05	1.4690395097D+06	5.3272612722D+06	-3.0111689081D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	1.3914050495D+06	5.3480633792D+06	-3.0111689081D+06
6.6700000000D+05	1.3134761969D+06	5.3677341774D+06	-3.0111689081D+06
6.6700000000D+05	1.2352694965D+06	5.3862695059D+06	-3.0111689081D+06
6.6700000000D+05	1.1568014921D+06	5.4036054437D+06	-3.0111689081D+06
6.6700000000D+05	1.0780887822D+06	5.4199183109D+06	-3.0111689081D+06
6.6700000000D+05	9.9914801766D+05	5.4350240695D+06	-3.0111689081D+06
6.6700000000D+05	9.1999589718D+05	5.4489813239D+06	-3.0111689081D+06
6.6700000000D+05	8.4064916430D+05	5.4617653219D+06	-3.0111689081D+06
-6.6700000000D+05	7.6112400374D+05	5.4734339548D+06	-3.0111689081D+06
6.6700000000D+05	6.8143903782D+05	5.4839247585D+06	-3.0111689081D+06
-6.6700000000D+05	6.0160932291D+05	5.4932555140D+06	-3.0111689081D+06
0.0	5.2165234588D+05	5.5014242474D+06	-3.0111689081D+06
6.6700000000D+05	4.4158562053D+05	5.5084292307D+06	-3.0111689081D+06
6.6700000000D+05	3.6142428398D+05	5.5142689821D+06	-3.0111689081D+06
6.6700000000D+05	2.8118709315D+05	5.5189422662D+06	-3.0111689081D+06
-6.6700000000D+05	2.0089042105D+05	5.5224480947D+06	-3.0111689081D+06
-6.6700000000D+05	1.2055125338D+05	5.5247857257D+06	-3.0111689081D+06
-6.6700000000D+05	4.0186584765D+04	5.5259546646D+06	-3.0111689081D+06
6.6700000000D+05	2.7062316516D+06	4.7670574349D+06	-3.0908064923D+06
6.6700000000D+05	2.6308138255D+06	4.8059123886D+06	-3.0908064923D+06
0.0	2.5564382958D+06	4.8437507181D+06	-3.0908064923D+06
-6.6700000000D+05	2.4957197939D+06	4.8805044192D+06	-3.0908064923D+06
-6.6700000000D+05	2.4244733970D+06	4.9163457046D+06	-3.0908064923D+06
-6.6700000000D+05	2.3527141364D+06	4.9510870051D+06	-3.0908064923D+06
-6.6700000000D+05	2.2804571917D+06	4.9847809718D+06	-3.0908064923D+06
-6.6700000000D+05	2.2077178478D+06	5.0174204771D+06	-3.0908064923D+06
-6.6700000000D+05	2.1345114919D+06	5.0489980166D+06	-3.0908064923D+06
-6.6700000000D+05	2.0608538096D+06	5.0795087104D+06	-3.0908064923D+06
-6.6700000000D+05	1.9867597823D+06	5.1089443046D+06	-3.0908064923D+06
6.6700000000D+05	1.9122456834D+06	5.1372991723D+06	-3.0908064923D+06
6.6700000000D+05	1.8373270755D+06	5.1645673156D+06	-3.0908064923D+06
0.0	1.7620198065D+06	5.1907429662D+06	-3.0908064923D+06
-6.6700000000D+05	1.6863398607D+06	5.2158205870D+06	-3.0908064923D+06
-6.6700000000D+05	1.6103030850D+06	5.2397948733D+06	-3.0908064923D+06
6.6700000000D+05	1.5339257260D+06	5.2626607535D+06	-3.0908064923D+06
-6.6700000000D+05	1.4572238562D+06	5.2844133908D+06	-3.0908064923D+06
6.6700000000D+05	1.3802137509D+06	5.3050481837D+06	-3.0908064923D+06
6.6700000000D+05	1.3029117306D+06	5.3245607670D+06	-3.0908064923D+06
0.0	1.2253340573D+06	5.3429470134D+06	-3.0908064923D+06
6.6700000000D+05	1.1474971815D+06	5.3602030333D+06	-3.0908064923D+06
-6.6700000000D+05	1.0694175867D+06	5.3763251765D+06	-3.0908064923D+06
-6.6700000000D+05	9.9111173532D+05	5.3913100326D+06	-3.0908064923D+06
6.6700000000D+05	9.1259624602D+05	5.4051544318D+06	-3.0908064923D+06
0.0	8.3382770962D+05	5.4178554454D+06	-3.0908064923D+06
6.6700000000D+05	7.5500277584D+05	5.4294103868D+06	-3.0908064923D+06
6.6700000000D+05	6.7595813160D+05	5.4398168116D+06	-3.0908064923D+06
-6.6700000000D+05	5.9677049794D+05	5.4490725186D+06	-3.0908064923D+06
6.6700000000D+05	5.1745662566D+05	5.4571755497D+06	-3.0908064923D+06
-6.6700000000D+05	4.3803329261D+05	5.4641241910D+06	-3.0908064923D+06
-6.6700000000D+05	3.5851729969D+05	5.4699169724D+06	-3.0908064923D+06
-6.6700000000D+05	2.7692546739D+05	5.4745526888D+06	-3.0908064923D+06
-6.6700000000D+05	1.9927463228D+05	5.4780302993D+06	-3.0908064923D+06
6.6700000000D+05	1.1952164338D+05	5.4803491285D+06	-3.0908064923D+06
-6.6700000000D+05	3.9863358641D+04	5.4815086657D+06	-3.0908064923D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	2.6837162202D+06	4.7273962497D+06	-3.1697992662D+06
-6.6700000000D+05	2.6146776015D+06	4.7659279361D+06	-3.1697992662D+06
6.6700000000D+05	2.5450658839D+06	4.8034514365D+06	-3.1697992662D+06
-6.6700000000D+05	2.4749557887D+06	4.8399588733D+06	-3.1697992662D+06
6.6700000000D+05	2.4043021559D+06	4.8754424639D+06	-3.1697992662D+06
6.6700000000D+05	2.3331399163D+06	4.9098947221D+06	-3.1697992662D+06
-6.6700000000D+05	2.2614841383D+06	4.9433083602D+06	-3.1697992662D+06
6.6700000000D+05	2.1893499746D+06	4.9756763098D+06	-3.1697992662D+06
-6.6700000000D+05	2.1167526843D+06	5.0069917241D+06	-3.1697992662D+06
6.6700000000D+05	2.0437076243D+06	5.0372479786D+06	-3.1697992662D+06
6.6700000000D+05	1.9702302462D+06	5.0664386731D+06	-3.1697992662D+06
6.6700000000D+05	1.8963360932D+06	5.0945576327D+06	-3.1697992662D+06
-6.6700000000D+05	1.8220407966D+06	5.1215989092D+06	-3.1697992662D+06
-6.6700000000D+05	1.7473600725D+06	5.1475567825D+06	-3.1697992662D+06
-6.6700000000D+05	1.6723097186D+06	5.1724257613D+06	-3.1697992662D+06
-6.6700000000D+05	1.5969056108D+06	5.1962005852D+06	-3.1697992662D+06
6.6700000000D+05	1.5211636997D+06	5.2188762249D+06	-3.1697992662D+06
-6.6700000000D+05	1.4451000074D+06	5.2404478835D+06	-3.1697992662D+06
0.0	1.3687306245D+06	5.2609109981D+06	-3.1697992662D+06
-6.6700000000D+05	1.2920717652D+06	5.2802612395D+06	-3.1697992662D+06
6.6700000000D+05	1.2151394662D+06	5.2984945154D+06	-3.1697992662D+06
0.0	1.1379501813D+06	5.3156069676D+06	-3.1697992662D+06
6.6700000000D+05	1.0605261738D+06	5.3315949773D+06	-3.1697992662D+06
-6.6700000000D+05	9.8286583860D+05	5.3464551617D+06	-3.1697992662D+06
6.6700000000D+05	9.0500358550D+05	5.3601843776D+06	-3.1697992662D+06
6.6700000000D+05	8.2694989204D+05	5.3727797205D+06	-3.1697992662D+06
6.6700000000D+05	7.4672126878D+05	5.3842385266D+06	-3.1697992662D+06
6.6700000000D+05	6.7033426390D+05	5.3945583718D+06	-3.1697992662D+06
-6.6700000000D+05	5.9166545911D+05	5.4037370726D+06	-3.1697992662D+06
-6.6700000000D+05	5.1315146606D+05	5.4117726876D+06	-3.1697992662D+06
6.6700000000D+05	4.3438892294D+05	5.4186035174D+06	-3.1697992662D+06
-6.6700000000D+05	3.5553449085D+05	5.4244081039D+06	-3.1697992662D+06
-6.6700000000D+05	2.7660465037D+05	5.4290052319D+06	-3.1697992662D+06
-6.6700000000D+05	1.9761669797D+05	5.4324539292D+06	-3.1697992662D+06
-6.6700000000D+05	1.1659674249D+05	5.4347534660D+06	-3.1697992662D+06
6.6700000000D+05	3.9531701623D+04	5.4359033560D+06	-3.1697992662D+06
6.6700000000D+05	2.6606339030D+06	4.6867349035D+06	-3.2481305219D+06
0.0	2.5921682002D+06	4.7249351707D+06	-3.2481305219D+06
6.6700000000D+05	2.5231950559D+06	4.7621359433D+06	-3.2481305219D+06
6.6700000000D+05	2.4536681647D+06	4.7983293520D+06	-3.2481305219D+06
6.6700000000D+05	2.3836222340D+06	4.8335077406D+06	-3.2481305219D+06
-6.6700000000D+05	2.3130720609D+06	4.8676636676D+06	-3.2481305219D+06
-6.6700000000D+05	2.2420326296D+06	4.9007899078D+06	-3.2481305219D+06
-6.6700000000D+05	2.1705169073D+06	4.9328794539D+06	-3.2481305219D+06
6.6700000000D+05	2.0985460418D+06	4.9639255175D+06	-3.2481305219D+06
6.6700000000D+05	2.0261292579D+06	4.9939215315D+06	-3.2481305219D+06
-6.6700000000D+05	1.9532838745D+06	5.0228611506D+06	-3.2481305219D+06
-6.6700000000D+05	1.8800253010D+06	5.0507382530D+06	-3.2481305219D+06
-6.6700000000D+05	1.8063690341D+06	5.0775409417D+06	-3.2481305219D+06
6.6700000000D+05	1.7323306549D+06	5.1032815457D+06	-3.2481305219D+06
6.6700000000D+05	1.6579258251D+06	5.1279366212D+06	-3.2481305219D+06
6.6700000000D+05	1.5831702841D+06	5.1515069527D+06	-3.2481305219D+06
-6.6700000000D+05	1.5082798454D+06	5.1739875543D+06	-3.2481305219D+06
-6.6700000000D+05	1.4326703932D+06	5.1953736706D+06	-3.2481305219D+06

MASSES(I)	X	Y	Z
6.6700000000D+05	1.3569578795D+06	5.2156607775D+06	-3.2481305219D+06
-6.6700000000D+05	1.2809533202D+06	5.2348445836D+06	-3.2481305219D+06
6.6700000000D+05	1.2046877918D+06	5.2529210309D+06	-3.2481305219D+06
6.6700000000D+05	1.1281624285D+06	5.2698862955D+06	-3.2481305219D+06
6.6700000000D+05	1.0512948418D+06	5.2857367887D+06	-3.2481305219D+06
-6.6700000000D+05	9.7441199873D+05	5.3004691575D+06	-3.2481305219D+06
6.6700000000D+05	8.9721945612D+05	5.3140802854D+06	-3.2481305219D+06
6.6700000000D+05	8.1983711916D+05	5.3265672933D+06	-3.2481305219D+06
6.6700000000D+05	7.4228135702D+05	5.3379275397D+06	-3.2481305219D+06
6.6700000000D+05	6.6456657554D+05	5.3481586215D+06	-3.2481305219D+06
6.6700000000D+05	5.8671521379D+05	5.3572553744D+06	-3.2481305219D+06
-6.6700000000D+05	5.0873774056D+05	5.3652248736D+06	-3.2481305219D+06
-6.6700000000D+05	4.3065265091D+05	5.3720564337D+06	-3.2481305219D+06
-6.6700000000D+05	3.5247646266D+05	5.3777516097D+06	-3.2481305219D+06
-6.6700000000D+05	2.7422571290D+05	5.3823091969D+06	-3.2481305219D+06
6.6700000000D+05	1.9591695448D+05	5.3857282311D+06	-3.2481305219D+06
-6.6700000000D+05	1.1756675255D+05	5.3880079892D+06	-3.2481305219D+06
-6.6700000000D+05	3.9191681003D+04	5.3891479867D+06	-3.2481305219D+06
-6.6700000000D+05	2.6367866885D+06	4.6450316551D+06	-3.3257836747D+06
6.6700000000D+05	2.5691501874D+06	4.6829424183D+06	-3.3257836747D+06
6.6700000000D+05	2.5007702181D+06	4.7198125699D+06	-3.3257836747D+06
-6.6700000000D+05	2.4318612455D+06	4.7556643105D+06	-3.3257836747D+06
6.6700000000D+05	2.3624378403D+06	4.7905500520D+06	-3.3257836747D+06
6.6700000000D+05	2.2925147062D+06	4.8244024190D+06	-3.3257836747D+06
6.6700000000D+05	2.2221066163D+06	4.8572342505D+06	-3.3257836747D+06
6.6700000000D+05	2.1512284705D+06	4.8890386014D+06	-3.3257836747D+06
-6.6700000000D+05	2.0798952622D+06	4.9198087439D+06	-3.3257836747D+06
-6.6700000000D+05	2.0081220809D+06	4.9495381090D+06	-3.3257836747D+06
-6.6700000000D+05	1.9359241091D+06	4.9782205879D+06	-3.3257836747D+06
-6.6700000000D+05	1.8633166194D+06	5.0058499332D+06	-3.3257836747D+06
6.6700000000D+05	1.7903149709D+06	5.0324203003D+06	-3.3257836747D+06
-6.6700000000D+05	1.7169346061D+06	5.0579262486D+06	-3.3257836747D+06
0.0	1.6431910476D+06	5.0823622027D+06	-3.3257836747D+06
6.6700000000D+05	1.5690998947D+06	5.1057230535D+06	-3.3257836747D+06
-6.6700000000D+05	1.4946708205D+06	5.1280038592D+06	-3.3257836747D+06
-6.6700000000D+05	1.4199375681D+06	5.1491999069D+06	-3.3257836747D+06
6.6700000000D+05	1.3448979476D+06	5.1693067126D+06	-3.3257836747D+06
-6.6700000000D+05	1.2695738526D+06	5.1883250231D+06	-3.3257836747D+06
6.6700000000D+05	1.1939811568D+06	5.2062358163D+06	-3.3257836747D+06
6.6700000000D+05	1.1181359109D+06	5.2230530025D+06	-3.3257836747D+06
6.6700000000D+05	1.0420541388D+06	5.2387599248D+06	-3.3257836747D+06
6.6700000000D+05	9.6575193456D+05	5.2533613600D+06	-3.3257836747D+06
-6.6700000000D+05	8.8924543889D+05	5.2668515194D+06	-3.3257836747D+06
-6.6700000000D+05	8.1255083567D+05	5.2792275493D+06	-3.3257836747D+06
-6.6700000000D+05	7.3560434857D+05	5.2904866318D+06	-3.3257836747D+06
6.6700000000D+05	6.5860223765D+05	5.3006269851D+06	-3.3257836747D+06
6.6700000000D+05	5.8150079585D+05	5.3096458641D+06	-3.3257836747D+06
6.6700000000D+05	5.0421634562D+05	5.3175415612D+06	-3.3257836747D+06
-6.6700000000D+05	4.2682523541D+05	5.3243124060D+06	-3.3257836747D+06
-6.6700000000D+05	3.4934353623D+05	5.3299569663D+06	-3.3257836747D+06
6.6700000000D+05	2.7178853820D+05	5.3344740480D+06	-3.3257836747D+06
-6.6700000000D+05	1.9417574707D+05	5.3378626956D+06	-3.3257836747D+06
-6.6700000000D+05	1.1652138075D+05	5.3401221924D+06	-3.3257836747D+06
-6.6700000000D+05	3.8843365842D+04	5.3412520602D+06	-3.3257836747D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	2.6127820847D+06	4.6024449738D+06	-3.4027422660D+06
-6.6700000000D+05	2.5455682472D+06	4.6399582173D+06	-3.4027422660D+06
-6.6700000000D+05	2.4778159299D+06	4.6764899419D+06	-3.4027422660D+06
-6.6700000000D+05	2.4095594650D+06	4.7120324198D+06	-3.4027422660D+06
-6.6700000000D+05	2.3407532954D+06	4.7465781326D+06	-3.4027422660D+06
6.6700000000D+05	2.2714719719D+06	4.7801197726D+06	-3.4027422660D+06
-6.6700000000D+05	2.2017101499D+06	4.8126502444D+06	-3.4027422660D+06
6.6700000000D+05	2.1314825867D+06	4.8441626667D+06	-3.4027422660D+06
0.0	2.0608041379D+06	4.8746503736D+06	-3.4027422660D+06
-6.6700000000D+05	1.9896897545D+06	4.9041069156D+06	-3.4027422660D+06
6.6700000000D+05	1.9181544798D+06	4.9325260619D+06	-3.4027422660D+06
6.6700000000D+05	1.8462134461D+06	4.9599018005D+06	-3.4027422660D+06
-6.6700000000D+05	1.7738818715D+06	4.9862283407D+06	-3.4027422660D+06
6.6700000000D+05	1.7011750567D+06	5.0115001133D+06	-3.4027422660D+06
6.6700000000D+05	1.6281083820D+06	5.0357117725D+06	-3.4027422660D+06
-6.6700000000D+05	1.5546973035D+06	5.0588581966D+06	-3.4027422660D+06
6.6700000000D+05	1.4809573503D+06	5.0809344894D+06	-3.4027422660D+06
-6.6700000000D+05	1.4069041211D+06	5.1019359809D+06	-3.4027422660D+06
-6.6700000000D+05	1.3325532809D+06	5.1218582255D+06	-3.4027422660D+06
6.6700000000D+05	1.2579205574D+06	5.1406970179D+06	-3.4027422660D+06
6.6700000000D+05	1.1830217383D+06	5.1584483642D+06	-3.4027422660D+06
-6.6700000000D+05	1.1078726073D+06	5.1751085121D+06	-3.4027422660D+06
-6.6700000000D+05	1.0324892412D+06	5.1906739375D+06	-3.4027422660D+06
-6.6700000000D+05	9.5688740032D+05	5.2051413478D+06	-3.4027422660D+06
6.6700000000D+05	8.8106315516D+05	5.2185076825D+06	-3.4027422660D+06
-6.6700000000D+05	8.0509252307D+05	5.2307701143D+06	-3.4027422660D+06
6.6700000000D+05	7.2893152480D+05	5.2419260491D+06	-3.4027422660D+06
6.6700000000D+05	6.5261645116D+05	5.2519731270D+06	-3.4027422660D+06
-6.6700000000D+05	5.7616326555D+05	5.2609092228D+06	-3.4027422660D+06
6.6700000000D+05	4.9950820058D+05	5.2687324461D+06	-3.4027422660D+06
6.6700000000D+05	4.2290745464D+05	5.2754411421D+06	-3.4027422660D+06
-6.6700000000D+05	3.4613724850D+05	5.2810338916D+06	-3.4027422660D+06
6.6700000000D+05	2.6929332182D+05	5.2855095115D+06	-3.4027422660D+06
6.6700000000D+05	1.9239342976D+05	5.2888670552D+06	-3.4027422660D+06
6.6700000000D+05	1.1545233953D+05	5.2911058122D+06	-3.4027422660D+06
6.6700000000D+05	3.8488026982D+04	5.2922253092D+06	-3.4027422660D+06
-6.6700000000D+05	2.5580241185D+06	4.5588335384D+06	-3.4789899674D+06
6.6700000000D+05	2.5214471798D+06	4.5959913173D+06	-3.4789899674D+06
-6.6700000000D+05	2.4543306639D+06	4.6321768778D+06	-3.4789899674D+06
6.6700000000D+05	2.3867073670D+06	4.6673825656D+06	-3.4789899674D+06
6.6700000000D+05	2.3185729952D+06	4.7016009332D+06	-3.4789899674D+06
-6.6700000000D+05	2.2499481614D+06	4.7348247422D+06	-3.4789899674D+06
-6.6700000000D+05	2.1808473823D+06	4.7670469647D+06	-3.4789899674D+06
6.6700000000D+05	2.1112852751D+06	4.7982607845D+06	-3.4789899674D+06
6.6700000000D+05	2.0412765548D+06	4.8284595986D+06	-3.4789899674D+06
6.6700000000D+05	1.9708360307D+06	4.8576370191D+06	-3.4789899674D+06
-6.6700000000D+05	1.8999786035D+06	4.8857868737D+06	-3.4789899674D+06
-6.6700000000D+05	1.8287192622D+06	4.9129032078D+06	-3.4789899674D+06
6.6700000000D+05	1.7570730807D+06	4.9389802852D+06	-3.4789899674D+06
6.6700000000D+05	1.6850552147D+06	4.9640125899D+06	-3.4789899674D+06
0.0	1.6126808980D+06	4.9879948264D+06	-3.4789899674D+06
-6.6700000000D+05	1.5399694422D+06	5.0109219217D+06	-3.4789899674D+06
-6.6700000000D+05	1.4669242275D+06	5.0327890260D+06	-3.4789899674D+06
-6.6700000000D+05	1.3935727052D+06	5.0535915134D+06	-3.4789899674D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	1.3199263920D+06	5.0733249836D+06	-3.4789899674D+06
-6.6700000000D+05	1.2460008666D+06	5.0919852621D+06	-3.4789899674D+06
0.0	1.1718117670D+06	5.1095684017D+06	-3.4789899674D+06
6.6700000000D+05	1.0973747688D+06	5.1260706829D+06	-3.4789899674D+06
6.6700000000D+05	1.0227056722D+06	5.1414886149D+06	-3.4789899674D+06
6.6700000000D+05	9.4782021844D+05	5.1558189362D+06	-3.4789899674D+06
0.0	8.7273426641D+05	5.1690586154D+06	-3.4789899674D+06
-6.6700000000D+05	7.9748369953D+05	5.1812048519D+06	-3.4789899674D+06
-6.6700000000D+05	7.2202444025D+05	5.1922550762D+06	-3.4789899674D+06
0.0	6.4643244671D+05	5.2022069513D+06	-3.4789899674D+06
-6.6700000000D+05	5.7070370934D+05	5.2110583709D+06	-3.4789899674D+06
6.6700000000D+05	4.9485424750D+05	5.2188074637D+06	-3.4789899674D+06
6.6700000000D+05	4.1890106110D+05	5.2254525900D+06	-3.4789899674D+06
6.6700000000D+05	3.4285735220D+05	5.2309923442D+06	-3.4789899674D+06
-6.6700000000D+05	2.6674207157D+05	5.2354255545D+06	-3.4789899674D+06
-6.6700000000D+05	1.9057036535D+05	5.2387512830D+06	-3.4789899674D+06
-6.6700000000D+05	1.1435834660D+05	5.2409688262D+06	-3.4789899674D+06
6.6700000000D+05	3.8122136938D+04	5.2420777151D+06	-3.4789899674D+06
-6.6700000000D+05	2.5627178351D+06	4.5142562361D+06	-3.5545105836D+06
-6.6700000000D+05	2.4967919007D+06	4.5510506778D+06	-3.5545105836D+06
6.6700000000D+05	2.4303378040D+06	4.5868824073D+06	-3.5545105836D+06
-6.6700000000D+05	2.3633896042D+06	4.6217438464D+06	-3.5545105836D+06
0.0	2.2959014657D+06	4.6556276191D+06	-3.5545105836D+06
6.6700000000D+05	2.2279476610D+06	4.6885265583D+06	-3.5545105836D+06
6.6700000000D+05	2.1595225649D+06	4.7204337046D+06	-3.5545105836D+06
6.6700000000D+05	2.0906406517D+06	4.7513423086D+06	-3.5545105836D+06
0.0	2.0213104924D+06	4.7812458319D+06	-3.5545105836D+06
-6.6700000000D+05	1.9515647516D+06	4.8101379490D+06	-3.5545105836D+06
-6.6700000000D+05	1.8814001843D+06	4.8380125480D+06	-3.5545105836D+06
6.6700000000D+05	1.8108370329D+06	4.8648037324D+06	-3.5545105836D+06
6.6700000000D+05	1.7398920239D+06	4.8906858223D+06	-3.5545105836D+06
6.6700000000D+05	1.6685783648D+06	4.9154733555D+06	-3.5545105836D+06
6.6700000000D+05	1.5969117411D+06	4.9392210883D+06	-3.5545105836D+06
6.6700000000D+05	1.5249073129D+06	4.9619239973D+06	-3.5545105836D+06
6.6700000000D+05	1.4525803116D+06	4.9835772800D+06	-3.5545105836D+06
6.6700000000D+05	1.3799460371D+06	5.0041763561D+06	-3.5545105836D+06
-6.6700000000D+05	1.3070149541D+06	5.0237166879D+06	-3.5545105836D+06
0.0	1.2335171892D+06	5.0421946820D+06	-3.5545105836D+06
-6.6700000000D+05	1.1603535273D+06	5.0596058897D+06	-3.5545105836D+06
-6.6700000000D+05	1.0868444087D+06	5.0759468079D+06	-3.5545105836D+06
-6.6700000000D+05	1.0127054256D+06	5.0912139799D+06	-3.5545105836D+06
-6.6700000000D+05	9.3855221862D+05	5.1054341760D+06	-3.5545105836D+06
-6.6700000000D+05	8.6420047396D+05	5.1185143947D+06	-3.5545105836D+06
-6.6700000000D+05	7.8966571968D+05	5.1305418629D+06	-3.5545105836D+06
-6.6700000000D+05	7.1496432247D+05	5.1414840353D+06	-3.5545105836D+06
-6.6700000000D+05	6.4011148448D+05	5.1513385984D+06	-3.5545105836D+06
-6.6700000000D+05	5.6512323978D+05	5.1601034671D+06	-3.5545105836D+06
-6.6700000000D+05	4.9001545109D+05	5.1677767875D+06	-3.5545105836D+06
6.6700000000D+05	4.1480400641D+05	5.1743569363D+06	-3.5545105836D+06
6.6700000000D+05	3.3950481570D+05	5.1798425215D+06	-3.5545105836D+06
6.6700000000D+05	2.6413380744D+05	5.1842323829D+06	-3.5545105836D+06
6.6700000000D+05	1.8870692534D+05	5.1875255917D+06	-3.5545105836D+06
6.6700000000D+05	1.1324012490D+05	5.1897214513D+06	-3.5545105836D+06
6.6700000000D+05	3.7749370087D+04	5.1908194912D+06	-3.5545105836D+06

MASSES(I)	X	Y	Z
6.6700000000D+05	2.5368683973D+06	4.4687221612D+06	-3.6292880565D+06
-6.6700000000D+05	2.4716074399D+06	4.5051454674D+06	-3.6292880565D+06
-6.6700000000D+05	2.4058230481D+06	4.5406157725D+06	-3.6292880565D+06
-6.6700000000D+05	2.3395309377D+06	4.5751255732D+06	-3.6292880565D+06
6.6700000000D+05	2.2721433320D+06	4.6086675695D+06	-3.6292880565D+06
6.6700000000D+05	2.2054749589D+06	4.6412346680D+06	-3.6292880565D+06
6.6700000000D+05	2.1377400481D+06	4.6728199736D+06	-3.6292880565D+06
6.6700000000D+05	2.0695529280D+06	4.7034168109D+06	-3.6292880565D+06
0.0	2.0009230226D+06	4.7330187056D+06	-3.6292880565D+06
6.6700000000D+05	1.9318798487D+06	4.7616193957D+06	-3.6292880565D+06
-6.6700000000D+05	1.8624230124D+06	4.7892128312D+06	-3.6292880565D+06
6.6700000000D+05	1.7925722062D+06	4.8157931750D+06	-3.6292880565D+06
6.6700000000D+05	1.7223422063D+06	4.8413548046D+06	-3.6292880565D+06
6.6700000000D+05	1.6517478687D+06	4.8658923126D+06	-3.6292880565D+06
6.6700000000D+05	1.5808041267D+06	4.8894005085D+06	-3.6292880565D+06
6.6700000000D+05	1.5095259875D+06	4.9118744194D+06	-3.6292880565D+06
-6.6700000000D+05	1.4379285290D+06	4.9333092914D+06	-3.6292880565D+06
-6.6700000000D+05	1.3660268967D+06	4.9537005901D+06	-3.6292880565D+06
6.6700000000D+05	1.2938383003D+06	4.9730440021D+06	-3.6292880565D+06
-6.6700000000D+05	1.2213720107D+06	4.9913334356D+06	-3.6292880565D+06
-6.6700000000D+05	1.1486493566D+06	5.0085710212D+06	-3.6292880565D+06
-6.6700000000D+05	1.0756837221D+06	5.0247471133D+06	-3.6292880565D+06
-6.6700000000D+05	1.0024905414D+06	5.0398602891D+06	-3.6292880565D+06
-6.6700000000D+05	9.2908529765D+05	5.0539073520D+06	-3.6292880565D+06
-6.6700000000D+05	8.5548351881D+05	5.0668853320D+06	-3.6292880565D+06
6.6700000000D+05	7.8170077429D+05	5.0787914821D+06	-3.6292880565D+06
-6.6700000000D+05	7.0775267179D+05	5.0896232842D+06	-3.6292880565D+06
6.6700000000D+05	6.3365405403D+05	5.0993784469D+06	-3.6292880565D+06
-6.6700000000D+05	5.5942299536D+05	5.1080549068D+06	-3.6292880565D+06
-6.6700000000D+05	4.8507279856D+05	5.1156508285D+06	-3.6292880565D+06
-6.6700000000D+05	4.1061999121D+05	5.1221646051D+06	-3.6292880565D+06
0.0	3.3606032296D+05	5.1275945588D+06	-3.6292880565D+06
6.6700000000D+05	2.6140956156D+05	5.1319404403D+06	-3.6292880565D+06
-6.6700000000D+05	1.8680348991D+05	5.1352004318D+06	-3.6292880565D+06
6.6700000000D+05	1.1209790256D+05	5.1373741424D+06	-3.6292880565D+06
6.6700000000D+05	3.7368602457D+04	5.1384611127D+06	-3.6292880565D+06
-6.6700000000D+05	2.5104810846D+06	4.4222406137D+06	-3.7033064686D+06
-6.6700000000D+05	2.4458989410D+06	4.4582850618D+06	-3.7033064686D+06
-6.6700000000D+05	2.3807994014D+06	4.4933864214D+06	-3.7033064686D+06
6.6700000000D+05	2.3151962366D+06	4.5275372674D+06	-3.7033064686D+06
-6.6700000000D+05	2.2491033241D+06	4.5607303756D+06	-3.7033064686D+06
-6.6700000000D+05	2.1825346459D+06	4.5929587245D+06	-3.7033064686D+06
-6.6700000000D+05	2.1155042809D+06	4.6242154966D+06	-3.7033064686D+06
6.6700000000D+05	2.0480264112D+06	4.6544940800D+06	-3.7033064686D+06
-6.6700000000D+05	1.9801153099D+06	4.6837880697D+06	-3.7033064686D+06
6.6700000000D+05	1.9117853426D+06	4.7120912688D+06	-3.7033064686D+06
-6.6700000000D+05	1.8430509636D+06	4.7393976904D+06	-3.7033064686D+06
-6.6700000000D+05	1.7739267127D+06	4.7657015580D+06	-3.7033064686D+06
6.6700000000D+05	1.7044272122D+06	4.7909973075D+06	-3.7033064686D+06
-6.6700000000D+05	1.6345671637D+06	4.8152795878D+06	-3.7033064686D+06
-6.6700000000D+05	1.5643013452D+06	4.8385432625D+06	-3.7033064686D+06
6.6700000000D+05	1.4938246077D+06	4.8607834104D+06	-3.7033064686D+06
6.6700000000D+05	1.4229718723D+06	4.8819553269D+06	-3.7033064686D+06
-6.6700000000D+05	1.3518181270D+06	4.9021745250D+06	-3.7033064686D+06

MASSES(I)	X	Y	Z
6.6700000000D+05	1.2803784232D+06	4.9213167359D+06	-3.7033064686D+06
6.6700000000D+05	1.2086678731D+06	4.9394179105D+06	-3.7033064686D+06
-6.6700000000D+05	1.1367016401D+06	4.9564742196D+06	-3.7033064686D+06
6.6700000000D+05	1.0644949656D+06	4.9724820553D+06	-3.7033064686D+06
-6.6700000000D+05	9.9206310595D+05	4.9874380312D+06	-3.7033064686D+06
-6.6700000000D+05	9.1942138908D+05	5.0013389838D+06	-3.7033064686D+06
-6.6700000000D+05	8.4658518135D+05	5.0141819724D+06	-3.7033064686D+06
-6.6700000000D+05	7.7356969026D+05	5.0259642603D+06	-3.7033064686D+06
-6.6700000000D+05	7.0039096117D+05	5.0366334150D+06	-3.7033064686D+06
6.6700000000D+05	6.2706387407D+05	5.0463371092D+06	-3.7033064686D+06
-6.6700000000D+05	5.5360414030D+05	5.0549233206D+06	-3.7033064686D+06
-6.6700000000D+05	4.8002729925D+05	5.0624402331D+06	-3.7033064686D+06
0.0	4.0634691507D+05	5.0688862565D+06	-3.7033064686D+06
6.6700000000D+05	3.3258457341D+05	5.0742602720D+06	-3.7033064686D+06
6.6700000000D+05	2.5874937808D+05	5.0785604085D+06	-3.7033064686D+06
6.6700000000D+05	1.8486044781D+05	5.0817604906D+06	-3.7033064686D+06
6.6700000000D+05	1.1093191287D+05	5.0839375913D+06	-3.7033064686D+06
-6.6700000000D+05	3.6979911519D+04	5.0850132554D+06	-3.7033064686D+06
0.0	2.4835612928D+06	4.3748210982D+06	-3.7765500462D+06
0.0	2.4196716610D+06	4.4104790430D+06	-3.7765500462D+06
6.6700000000D+05	2.3552701812D+06	4.4452040121D+06	-3.7765500462D+06
-6.6700000000D+05	2.2903704766D+06	4.4789886599D+06	-3.7765500462D+06
-6.6700000000D+05	2.2249862759D+06	4.5118258397D+06	-3.7765500462D+06
-6.6700000000D+05	2.1591314102D+06	4.5437086653D+06	-3.7765500462D+06
6.6700000000D+05	2.0928198101D+06	4.5746302122D+06	-3.7765500462D+06
6.6700000000D+05	2.0260655030D+06	4.6045641195D+06	-3.7765500462D+06
-6.6700000000D+05	1.9588620098D+06	4.6335639908D+06	-3.7765500462D+06
6.6700000000D+05	1.8912653421D+06	4.6615636959D+06	-3.7765500462D+06
6.6700000000D+05	1.8232879992D+06	4.6885773117D+06	-3.7765500462D+06
6.6700000000D+05	1.7549049650D+06	4.7145991240D+06	-3.7765500462D+06
6.6700000000D+05	1.6861507049D+06	4.7396236281D+06	-3.7765500462D+06
6.6700000000D+05	1.6170397631D+06	4.7636455306D+06	-3.7765500462D+06
-6.6700000000D+05	1.5475867588D+06	4.7866597493D+06	-3.7765500462D+06
6.6700000000D+05	1.4778063841D+06	4.8086614174D+06	-3.7765500462D+06
6.6700000000D+05	1.4077133998D+06	4.8296458794D+06	-3.7765500462D+06
6.6700000000D+05	1.3373226333D+06	4.8496080960D+06	-3.7765500462D+06
6.6700000000D+05	1.2666439748D+06	4.8685456464D+06	-3.7765500462D+06
6.6700000000D+05	1.1957073742D+06	4.8864527227D+06	-3.7765500462D+06
6.6700000000D+05	1.1245125382D+06	4.9033201377D+06	-3.7765500462D+06
6.6700000000D+05	1.0530804272D+06	4.9191623220D+06	-3.7765500462D+06
-6.6700000000D+05	9.8142525154D+05	4.9339579256D+06	-3.7765500462D+06
-6.6700000000D+05	9.0956246899D+05	4.9477398188D+06	-3.7765500462D+06
-6.6700000000D+05	8.3750728111D+05	4.9604150925D+06	-3.7765500462D+06
6.6700000000D+05	7.6527493016D+05	4.9720710592D+06	-3.7765500462D+06
-6.6700000000D+05	6.9288024590D+05	4.9826752530D+06	-3.7765500462D+06
-6.6700000000D+05	6.2033959234D+05	4.9922254310D+06	-3.7765500462D+06
-6.6700000000D+05	5.4766786446D+05	5.0007195729D+06	-3.7765500462D+06
-6.6700000000D+05	4.7487998503D+05	5.0081558817D+06	-3.7765500462D+06
-6.6700000000D+05	4.0199165133D+05	5.0145327847D+06	-3.7765500462D+06
6.6700000000D+05	3.2901822185D+05	5.0198489320D+06	-3.7765500462D+06
-6.6700000000D+05	2.5597531312D+05	5.0241032011D+06	-3.7765500462D+06
-6.6700000000D+05	1.8287819636D+05	5.0272946901D+06	-3.7765500462D+06
6.6700000000D+05	1.0974239425D+05	5.0294227246D+06	-3.7765500462D+06
6.6700000000D+05	3.6583317651D+04	5.0304660544D+06	-3.7765500462D+06

MASSES(I)	X	Y	Z
6.6700000000D+05	2.4561145328D+06	4.3264733222D+06	-3.8490031636D+06
-6.6700000000D+05	2.3929309691D+06	4.3617371979D+06	-3.8490031636D+06
-6.6700000000D+05	2.3292412140D+06	4.3960784084D+06	-3.8490031636D+06
6.6700000000D+05	2.2650587402D+06	4.4294858895D+06	-3.8490031636D+06
-6.6700000000D+05	2.2003971246D+06	4.4619659734D+06	-3.8490031636D+06
6.6700000000D+05	2.1352700455D+06	4.4934943906D+06	-3.8490031636D+06
6.6700000000D+05	2.0696912796D+06	4.5240742714D+06	-3.8490031636D+06
6.6700000000D+05	2.0036746991D+06	4.5536971469D+06	-3.8490031636D+06
-6.6700000000D+05	1.9372342691D+06	4.5823567509D+06	-3.8490031636D+06
6.6700000000D+05	1.8703840644D+06	4.6100470208D+06	-3.8490031636D+06
6.6700000000D+05	1.8031381650D+06	4.6367620991D+06	-3.8490031636D+06
6.6700000000D+05	1.7355100572D+06	4.6624963347D+06	-3.8490031636D+06
6.6700000000D+05	1.6675164260D+06	4.6872442837D+06	-3.8490031636D+06
6.6700000000D+05	1.5991692549D+06	4.7110007111D+06	-3.8490031636D+06
-6.6700000000D+05	1.5304838017D+06	4.7337605916D+06	-3.8490031636D+06
6.6700000000D+05	1.4614745958D+06	4.7555191107D+06	-3.8490031636D+06
6.6700000000D+05	1.3921562352D+06	4.7762716655D+06	-3.8490031636D+06
-6.6700000000D+05	1.3225433833D+06	4.7960138663D+06	-3.8490031636D+06
6.6700000000D+05	1.2526507658D+06	4.8147415367D+06	-3.8490031636D+06
6.6700000000D+05	1.1824931670D+06	4.8324507153D+06	-3.8490031636D+06
-6.6700000000D+05	1.1120854284D+06	4.8491376559D+06	-3.8490031636D+06
6.6700000000D+05	1.0414424435D+06	4.8647968260D+06	-3.8490031636D+06
-6.6700000000D+05	9.7057915587D+05	4.8794309205D+06	-3.8490031636D+06
-6.6700000000D+05	8.9951055567D+05	4.8930308364D+06	-3.8490031636D+06
6.6700000000D+05	8.2825167648D+05	4.9055956994D+06	-3.8490031636D+06
-6.6700000000D+05	7.5681759213D+05	4.9171228516D+06	-3.8490031636D+06
-6.6700000000D+05	6.8522341350D+05	4.9276098546D+06	-3.8490031636D+06
0.0	6.1348428534D+05	4.9370544900D+06	-3.8490031636D+06
6.6700000000D+05	5.4161538309D+05	4.9454547600D+06	-3.8490031636D+06
-6.6700000000D+05	4.6963190960D+05	4.9528088874D+06	-3.8490031636D+06
-6.6700000000D+05	3.9754909200D+05	4.9591153168D+06	-3.8490031636D+06
-6.6700000000D+05	3.2538217838D+05	4.9643727140D+06	-3.8490031636D+06
-6.6700000000D+05	2.5314643468D+05	4.9685799670D+06	-3.8490031636D+06
-6.6700000000D+05	1.8085714136D+05	4.9717361857D+06	-3.8490031636D+06
6.6700000000D+05	1.0852959021D+05	4.9738407025D+06	-3.8490031636D+06
6.6700000000D+05	3.6179611131D+04	4.9748930722D+06	-3.8490031636D+06
6.6700000000D+05	2.4281464299D+06	4.2772071951D+06	-3.9206503459D+06
-6.6700000000D+05	2.3656625459D+06	4.3120695163D+06	-3.9206503459D+06
-6.6700000000D+05	2.3027178345D+06	4.3460196789D+06	-3.9206503459D+06
0.0	2.2392662151D+06	4.3790505013D+06	-3.9206503459D+06
6.6700000000D+05	2.1753409099D+06	4.4111549962D+06	-3.9206503459D+06
6.6700000000D+05	2.1109554415D+06	4.4423263724D+06	-3.9206503459D+06
-6.6700000000D+05	2.0461234297D+06	4.4725580360D+06	-3.9206503459D+06
-6.6700000000D+05	1.9808585889D+06	4.5018435919D+06	-3.9206503459D+06
-6.6700000000D+05	1.9151747248D+06	4.5301768452D+06	-3.9206503459D+06
6.6700000000D+05	1.8490857321D+06	4.5575518023D+06	-3.9206503459D+06
-6.6700000000D+05	1.7826055909D+06	4.5839626725D+06	-3.9206503459D+06
0.0	1.7157483642D+06	4.6094038688D+06	-3.9206503459D+06
-6.6700000000D+05	1.6485281947D+06	4.6338700097D+06	-3.9206503459D+06
-6.6700000000D+05	1.5809593019D+06	4.6573559195D+06	-3.9206503459D+06
6.6700000000D+05	1.5130559791D+06	4.6798560302D+06	-3.9206503459D+06
-6.6700000000D+05	1.4448325902D+06	4.7013673821D+06	-3.9206503459D+06
-6.6700000000D+05	1.3763035870D+06	4.7218836248D+06	-3.9206503459D+06
-6.6700000000D+05	1.3074834059D+06	4.7414310185D+06	-3.9206503459D+06

MASSES(I)	X	Y	Z
-6.6700000000D+05	1.2383866647D+06	4.7599154345D+06	-3.9206503459D+06
-6.6700000000D+05	1.1690279601D+06	4.7774229563D+06	-3.9206503459D+06
-6.6700000000D+05	1.0994219637D+06	4.7939198805D+06	-3.9206503459D+06
0.0	1.0295833999D+06	4.8094027174D+06	-3.9206503459D+06
6.6700000000D+05	9.5952704194D+05	4.8238681917D+06	-3.9206503459D+06
-6.6700000000D+05	8.8926770935D+05	4.8373152436D+06	-3.9206503459D+06
6.6700000000D+05	8.1882026449D+05	4.8497350288D+06	-3.9206503459D+06
6.6700000000D+05	7.4819560956D+05	4.8611309198D+06	-3.9206503459D+06
6.6700000000D+05	6.7742068336D+05	4.8714985059D+06	-3.9206503459D+06
6.6700000000D+05	6.0649845820D+05	4.8808355940D+06	-3.9206503459D+06
6.6700000000D+05	5.3544793069D+05	4.8891402089D+06	-3.9206503459D+06
0.0	4.6428414059D+05	4.8964105940D+06	-3.9206503459D+06
6.6700000000D+05	3.9302214761D+05	4.9026452112D+06	-3.9206503459D+06
-6.6700000000D+05	3.2167700825D+05	4.9078427417D+06	-3.9206503459D+06
0.0	2.5026382256D+05	4.9120020801D+06	-3.9206503459D+06
-6.6700000000D+05	1.7879769704D+05	4.9151223646D+06	-3.9206503459D+06
-6.6700000000D+05	1.0729374933D+05	4.9172029170D+06	-3.9206503459D+06
-6.6700000000D+05	3.5767105123D+04	4.9182433032D+06	-3.9206503459D+06
6.6700000000D+05	2.3990027234D+06	4.2270328263D+06	-3.9914762731D+06
6.6700000000D+05	2.3374213629D+06	4.2614861902D+06	-3.9914762731D+06
6.6700000000D+05	2.2757054854D+06	4.2950380955D+06	-3.9914762731D+06
6.6700000000D+05	2.2129951939D+06	4.3276814455D+06	-3.9914762731D+06
0.0	2.1498227734D+06	4.3594093342D+06	-3.9914762731D+06
6.6700000000D+05	2.0861925376D+06	4.3902150503D+06	-3.9914762731D+06
-6.6700000000D+05	2.0221210967D+06	4.4200920772D+06	-3.9914762731D+06
-6.6700000000D+05	1.9576218540D+06	4.4490340946D+06	-3.9914762731D+06
-6.6700000000D+05	1.8927085036D+06	4.4770349810D+06	-3.9914762731D+06
-6.6700000000D+05	1.8273947769D+06	4.5040886124D+06	-3.9914762731D+06
-6.6700000000D+05	1.7616944902D+06	4.5301898662D+06	-3.9914762731D+06
-6.6700000000D+05	1.6956215414D+06	4.5553326211D+06	-3.9914762731D+06
-6.6700000000D+05	1.6291699074D+06	4.5795117585D+06	-3.9914762731D+06
0.0	1.5624136408D+06	4.6027221636D+06	-3.9914762731D+06
-6.6700000000D+05	1.4953063673D+06	4.6249589266D+06	-3.9914762731D+06
0.0	1.4278837823D+06	4.6462173437D+06	-3.9914762731D+06
-6.6700000000D+05	1.3601586482D+06	4.6664929178D+06	-3.9914762731D+06
-6.6700000000D+05	1.2921457915D+06	4.6857813600D+06	-3.9914762731D+06
6.6700000000D+05	1.2238595992D+06	4.7040785901D+06	-3.9914762731D+06
6.6700000000D+05	1.1553145164D+06	4.7213807375D+06	-3.9914762731D+06
6.6700000000D+05	1.0865250428D+06	4.7376841423D+06	-3.9914762731D+06
6.6700000000D+05	1.0175057298D+06	4.7529893556D+06	-3.9914762731D+06
0.0	9.4827111765D+05	4.7672811407D+06	-3.9914762731D+06
6.6700000000D+05	8.7683603185D+05	4.7805684735D+06	-3.9914762731D+06
6.6700000000D+05	8.0921498046D+05	4.7928445433D+06	-3.9914762731D+06
6.6700000000D+05	7.3942275066D+05	4.8041067532D+06	-3.9914762731D+06
6.6700000000D+05	6.6947410661D+05	4.8143527209D+06	-3.9914762731D+06
6.6700000000D+05	5.9938334440D+05	4.8235802795D+06	-3.9914762731D+06
6.6700000000D+05	5.2916679083D+05	4.8317874754D+06	-3.9914762731D+06
6.6700000000D+05	4.5883779936D+05	4.8389725742D+06	-3.9914762731D+06
0.0	3.8841174711D+05	4.8451340554D+06	-3.9914762731D+06
6.6700000000D+05	3.1790331174D+05	4.8502706156D+06	-3.9914762731D+06
-6.6700000000D+05	2.4732806828D+05	4.8543811683D+06	-3.9914762731D+06
6.6700000000D+05	1.7670028599D+05	4.8574648439D+06	-3.9914762731D+06
6.6700000000D+05	1.0603512521D+05	4.8595209900D+06	-3.9914762731D+06
-6.6700000000D+05	3.5347534163D+04	4.8605491719D+06	-3.9914762731D+06

CHEBYSHEV POLYNOMIAL FINITE ELEMENT SOFTWARE

```
C      PROGRAM LOCALG(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE2,TAPE3) *
C-----------------------------------------------------------------------
C
C     SECTION IIA
C      LOCAL GRAVITY MODEL -- BASIS FUNCTIONS AND OBSERVATIONS
C
C      BY                    JOHN L. JUNKINS AND JOHN SAUNDERS
C
C      MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C             DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C             SCHOOL OF ENGINEERING
C             VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C             BLACKSBURG, VIRGINIA    24061
C
C      DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C-----------------------------------------------------------------------
C   FOR P,   SOME POINT IN SPACE
C   X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES
C   H,ALAM,APHI--ELLIPSOIDAL COORDINATES
C   RN--EAST-WEST RADIUS OF CURVATURE
C   NORDER--ORDER OF THE BASIS FUNCTIONS DESIRED
C   H--HEIGHT ABOVE REF. ELLIPSOID ALONG NORMAL TO P
C   ALAM--ANGLE LAMBDA, GEODETIC/GEOCENTRIC LONGITUDE (RADIANS) OF P
C   APHI--ANGLE PHI, GEODETIC LATITUDE (RADIANS) OF P
C
C   INPUTS
C
C   FROM CARDS
C   FORMAT 1
C   NORDER--ORDER OF POLYNOMIALS DESIRED
C   MOBSU,MOBSE,MOBSN--OBSERVATION GRID PATTERN FOR ONE CELL
C
C   THE CARD INPUT FOR THIS SAMPLE RUN IS AS FOLLOWS:
C          3              4              4              4
C
C   FORMAT 2
C   HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
C   HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
C
C   THE CARD INPUT FOR THIS SAMPLE RUN IS AS FOLLOWS:
C300000.                    1.                        1.
C 0.                        75.                       -30.
C 300000.                   76.                       -29.
C
C   FROM DISK
C   FILE 2
C   FMVALS--PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL CONSTANT AND THE
C      1080 POINT MASSES (-1.E19, 0., OR +1.E19)
C   POSITS--PRECOMPUTED EARTH-FIXED X,Y,Z COORDINATES OF THE 1080 POINT
C      MASSES
C
C   PROCESS
C
```

```
      C   THIS PROGRAM ACCEPTS CARD INPUT WHICH DIMENSIONS THE REGION TO BE
      C   MODELED, THE FINITE ELEMENT GRID PATTERN TO BE USED, AND THE ORDER
      C   OF THE BASIS FUNCTIONS THAT ARE TO BE USED IN THE MODELING.  BY
      C   ESTABLISHING A STANDARD FINITE ELEMENT SIZE THE ENTIRE AREA IS
      C   BROKEN DOWN INTO A NUMBER OF CELLS AND EACH CELL IS MODELED
      C   SEPARATELY BY THE SAME PROCEDURE.  THE INPUT SPECIFICATIONS AND
      C   FINITE ELEMENT CELL DATA ARE THEN WRITTEN INTO THE FIRST RECORD OF
      C   A SEQUENTIAL ACCESS FILE (FILE 3).  BY USING A STANDARD FINITE
      C   ELEMENT CELL THE BASIS FUNCTIONS ARE EVALUATED AT EACH GRID POINT
      C   FOR EACH ORDER OF THE POLYNOMIALS UP TO THE SPECIFIED ORDER.  SINCE
      C   EACH NORMALIZED CELL APPEARS THE SAME, THESE POLYNOMIALS NEED ONLY BE
      C   CALCULATED ONCE NO MATTER HOW MANY CELLS MAKE UP THE ENTIRE AREA.
      C   VALUES OF THE BASIS FUNCTIONS OF PREDETERMINED ORDERS ARE THEN
      C   MULTIPLIED TOGETHER TO FORM A MATRIX WHICH WILL BE USED AS A LEAST
      C   SQUARES MATRIX IN SECTION II(B).  THIS MATRIX IS WRITTEN INTO THE
      C   SEQUENTIAL ACCESS FILE (FILE 3).    NEXT THE ACTUAL COORDINATES OF
      C   EACH GRID POINT OF THE CELL ARE DETERMINED AND THE THREE GRAVITY
      C   DISTURBANCE COMPONENTS AT THE POINT ARE FOUND.  IN THIS SOFTWARE THE
      C   DISTURBANCES ARE GIVEN BY SUBROUTINE PIMASS WHICH USES MASS MODEL 310
      C   TO CALCULATE THE 'OBSERVATIONS'.  THE THREE RECTANGULAR (X,Y,Z) COM-
      C   PONENTS ARE THEN CONVERTED TO THEIR ELLIPSOIDAL (H,LAMDA,PHI) VALUES
      C   AND THESE ARE WRITTEN ONTO FILE 3.  THIS PROCESS IS REPEATED FOR EACH
      C   CELL UNTIL ALL COMPONENTS OF ALL THE OBSERVATION POINTS HAVE BEEN
      C   RECORDED.
      C
      C   OUTPUTS
      C
      C   TO DISK
      C   FILE 3
      C   HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
      C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
      C   HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
      C   NH,NLAM,NPHI--NUMBER OF CELLS IN H,LAMDA,PHI DIRECTION
      C   NORDER--ORDER OF POLYNOMIALS DESIRED
      C   NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
      C   A--BASIS FUNCTION MATRIX FOR LEAST SQUARES FIT
      C   DELGU,DELGE,DELGN--GRAVITY DISTURBANCE COMPONENTS IN H,LAMDA,PHI
      C
      C   THE CREATED FILE 3 IS USED BY PROGRAM FINEG IN SECTION II(B) OF THIS
      C   SYSTEM OF PROGRAMS.  FINEG USES THE LEAST SQUARES MATRIX A AND THE
      C   GRAVITY DISTURBANCE OBSERVATIONS OF EACH CELL TO DETERMINE THE
      C   MODELING COEFFICIENTS WHICH CAN BE USED TO DESCRIBE THE FINITE
      C   ELEMENT CELLS SEPARATELY.  WHEN ALL OF THE CELLS HAVE BEEN MODELED
      C   THE ENTIRE REGION ITSELF IS MODELED.
      C
      C-----------------------------------------------------------------------
0001          IMPLICIT REAL*8 (A-H,O-Z)
0002          COMMON  /XYZ/X,Y,Z,GX,GY,GZ
0003          COMMON  /MASPOS/PMVALS(1080),POSITS(1080,3)
      C
0004          DIMENSION IX(7,7),IY(7,7),IZ(7,7),A(343,84)
0005          DIMENSION DELGU(343),DELGE(343),DELGN(343)
0006          DIMENSION  IX(84),IY(84),IZ(84)
0007          DIMENSION ICLL(6)
      C
```

```
0008            DATA RA/6378160.D0/
0009            DATA RB/6356774.504D0/
0010            DATA ICOL/4,10,20,35,56,84/
0011            DATA IX/1,1,1,2,1,1,1,2,2,3,1,1,1,1,2,2,2,3,3,4,1,1,1,1,1,2,2,2,2,
               +3,3,3,4,4,5,1,1,1,1,1,1,2,2,2,2,2,3,3,3,3,4,4,4,5,5,6,1,1,1,1,
               +1,1,1,2,2,2,2,2,2,3,3,3,3,3,4,4,4,4,5,5,5,6,6,7/
0012            DATA IY/1,1,2,1,1,2,3,1,2,1,1,2,3,4,1,2,3,1,2,1,1,2,3,4,5,1,2,3,4,
               +1,2,3,1,2,1,1,2,3,4,5,6,1,2,3,4,5,1,2,3,4,1,2,3,1,2,1,1,2,3,4,
               +5,6,7,1,2,3,4,5,6,1,2,3,4,5,1,2,3,4,1,2,3,1,2,1/
0013            DATA IZ/1,2,1,1,3,2,1,2,1,1,4,3,2,1,3,2,1,2,1,1,5,4,3,2,1,4,3,2,1,
               +3,2,1,2,1,1,6,5,4,3,2,1,5,4,3,2,1,4,3,2,1,3,2,1,2,1,1,7,6,5,4,
               +3,2,1,6,5,4,3,2,1,5,4,3,2,1,4,3,2,1,3,2,1,2,1,1/
          C
0014          1 FORMAT(8I10)
0015          2 FORMAT(3E20.14)
0016          3 FORMAT(1H1)
0017          4 FORMAT(1X,12I10)
0018          5 FORMAT(1X,3E20.12)
          C
          C   SET CONSTANTS
          C
0019            PI=DARCOS(-1.D0)
0020            DEGRAD=PI/180.D0
0021            A2=RA*RA
0022            B2=RB*RB
          C
          C   READ THE MASS VALUES AND THEIR X,Y,Z COORDINATES FROM FILE 2
          C
0023            READ(2) PMVALS,POSITS
          C
          C   READ THE POLYNOMIALS ORDER,GRID PATTERN,CELL SIZE,AND FINITE
          C   ELEMENT FIELD BOUNDS
          C
0024            READ(5,1,END=500) NORDER,MOBSU,MOBSE,MOBSN
0025            READ(5,2,END=500) HCELL,ALCELL,APCELL,HMIN,ALMIN,APMIN,HMAX,
               +ALMAX,APMAX
          C
          C   CALCULATE THE TOTAL NUMBER OF GRAVITY OBSERVATIONS TO BE MADE AND
          C   AND THE CONSTANTS NEEDED LATER IN THE PROGRAM
          C
0026            MOBS=MOBSU*MOBSE*MOBSN
0027            RMU=MOBSU
0028            RME=MOBSE
0029            RMN=MOBSN
0030            RMUM1=RMU-1.D0
0031            RMEM1=RME-1.D0
0032            RMNM1=RMN-1.D0
          C
          C   COMPUTE THE NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
          C
0033            NC=ICOL(NORDER)
          C
          C   OUTPUT THE INPUT AND OBSERVATION GRID DATA
          C
0034            WRITE(6,3)
```

```
      WRITE(6,4) NORDER,NC
      WRITE(6,4) MOBSU,MOBSE,MOBSN,MOBS
      WRITE(6,5) HCELL,ALCELL,APCELL,HMIN,ALMIN,APMIN,HMAX,ALMAX,APMAX
C
C  CONVERT ANGLES FROM DEGREES TO RADIANS
C
      ALCELL=ALCELL*DEGRAD
      APCELL=APCELL*DEGRAD
      ALMIN=ALMIN*DEGRAD
      APMIN=APMIN*DEGRAD
      ALMAX=ALMAX*DEGRAD
      APMAX=APMAX*DEGRAD
C
C  COMPUTE THE NUMBER OF FINAL CELLS IN UP,EASTERN, AND NORTHERN
C  DIRECTION AND THE TOTAL NUMBER OF FINAL CELLS IN THE FINITE
C  ELEMENT FIELD
C
      RNH=(HMAX-HMIN)/HCELL+.999D0
      RNE=(ALMAX-ALMIN)/ALCELL+.999D0
      RNP=(APMAX-APMIN)/APCELL+.999D0
      NH=RNH
      NLAM=RNE
      NPHI=RNP
      NCELLS=NH*NLAM*NPHI
C
C  COMPUTE THE INCREMENT SIZE WITHIN EACH CELL.
C
  110 DH=HCELL/KMUM1
      DLAM=ALCELL/KMEM1
      DPHI=APCELL/KMNM1
C
C  WRITE FIELD DATA ONTO FIRST RECORD OF SEQUENTIAL FILE
C
      WRITE(3) HMIN,HMAX,HCELL,ALMIN,ALMAX,ALCELL,APMIN,APMAX,APCELL,NH,
     +NLAM,NPHI,NORDER,NC,MOBS,NCELLS
C
C  EVALUATE THE BASIS FUNCTION AT EACH GRID POINT FOR EACH ORDER OF THE
C  FUNCTIONS UP TO THE NECESSARY ORDER
C
C  DETERMINE THE NORMALIZED H COORDINATE FOR EACH GRID POINT
C
      H=HMIN-DH
      DO 120 IH=1,MOBSU
      H=H+DH
      X1=(H-HMIN)/HCELL
      CALL CHEBY(X1,NORDER,TX(1,IH))
  120 CONTINUE
C
C  DETERMINE THE NORMALIZED LAMDA COODINATE FOR EACH GRID POINT
C
      ALAM=ALMIN-DLAM
      DO 130 IL=1,MOBSE
      ALAM=ALAM+DLAM
      X2=(ALAM-ALMIN)/ALCELL
      CALL CHEBY(X2,NORDER,TY(1,IL))
```

```
0066          130 CONTINUE
              C
              C  DETERMINE THE NORMALIZED PHI COORDINATE FOR EACH GRID POINT
              C
0067             APHI=APMIN-DPHI
0068             DO 140 IP=1,MOBSN
0069             APHI=APHI+DPHI
0070             X3=(APHI-APMIN)/APCELL
0071             CALL CHEBY(X3,NORDER,TZ(1,IP))
0072          140 CONTINUE
              C
              C-----------------------------------------------------------------------
              C
              C  NOTE:  IF A SYMMETRIC GRID PATTERN IS ALWAYS CHOSEN (SUCH AS 4*4*4)
              C  THE THREE SETS OF EVALUATIONS ABOVE ARE REDUNDANT AND TWO COULD BE
              C  ELIMINATED
              C
              C-----------------------------------------------------------------------
              C  FILL THE A MATRIX
              C
0073             DO 150  II=1,NC
0074             NX=IX(II)
0075             NY=IY(II)
0076             NZ=IZ(II)
0077             MJ=0
0078             DO 150 IH=1,MOBSU
0079             DO 150 IL=1,MOBSE
0080             DO 150 IP=1,MOBSN
0081             MJ=MJ+1
0082             A(MJ,II)=TX(NX,IH)*TY(NY,IL)*TZ(NZ,IP)
0083          150 CONTINUE
              C
              C  WRITE THE BASIS FUNCTION MATRIX TO DISK
              C
0084             WRITE(3) ((A(I,J),J=1,NC),I=1,MOBS)
              C
              C  GENERATE DATA FOR EACH CELL.
              C
0085             HMCELL=HMIN-HCELL
0086             DO 300 IHC=1,NH
              C
              C  INCREMENT THE H COORDINATE (HMCELL) 1 CELL SIZE
              C
0087             HMCELL=HMCELL+HCELL
              C
              C  RESET THE LAMDA COORDINATE TO ITS CELL MINIMUM VALUE
              C
0088             ALMCEL=ALMIN-ALCELL
0089             DO 300 ILC=1,NLAM
              C
              C  INCREMENT THE LAMDA COORDINATE (ALMCEL) 1 CELL SIZE
              C
0090             ALMCEL=ALMCEL+ALCELL
              C
              C  RESET THE PHI COORDINATE TO ITS CELL MINIMUM VALUE
```

```
                 C
 0091                  APMCEL=APMIN-APCELL
 0092                  DO 300 IPC=1,NPHI
                 C
                 C   INCREMENT THE PHI COORDINATE (APMCEL) 1 CELL SIZE
                 C
 0093                  APMCEL=APMCEL+APCELL
 0094                  MJ=0
 0095                  H=HMCELL-DH
 0096                  DO 250  IH=1,MOBSU
                 C
                 C   INCREMENT THE H COORDINATE (HMCELL) 1 GRID INCREMENT SIZE
 0097                  H=H+DH
 0098                  ALAM=ALMCEL-DLAM
 0099                  DO 250  IL=1,MOBSE
                 C
                 C
                 C   INCREMENT THE LAMDA COORDINATE (ALMCEL) 1 GRID INCREMENT SIZE
 0100                  ALAM=ALAM+DLAM
 0101                  COSL=DCOS(ALAM)
 0102                  SINL=DSIN(ALAM)
 0103                  APHI=APMCEL-DPHI
 0104                  DO 250  IP=1,MOBSN
                 C
                 C   INCREMENT THE PHI COORDINATE (APMCEL) 1 GRID INCREMENT SIZE
                 C
 0105                  APHI=APHI+DPHI
 0106                  COSP=DCOS(APHI)
 0107                  SINP=DSIN(APHI)
 0108                  MJ=MJ+1
                 C
                 C   TRANSFORM ALL ELLIPSOIDAL COORDINATES INTO X,Y,Z COORDINATES
                 C
 0109                   RN=DSQRT(A2*COSP*COSP + B2*SINP*SINP)
 0110                   ZN=B2/RN
 0111                   RN=A2/RN
 0112                   Y=(RN+H)*COSP
 0113                   X=Y*COSL
 0114                   Y=Y*SINL
 0115                   Z=(ZN+H)*SINP
                 C
                 C
                 C   OBTAIN THE GRAVITY 'OBSERVATION' AT THIS GRID POINT
                 C
 0116                  CALL PIMASS
                 C
                 C   DIRECTION COSINE MATRIX FOR 3-2 ROTATION (LAMBDA,-PHI) FROM X,Y,Z, TO
                 C   UP,EAST,NORTH
                 C
                 C   C11=COSP*COSL
                 C   C12=COSP*SINL
                 C   C13=SINP
                 C   C21=-SINL
                 C   C22=COSL
                 C   C23=0
```

```
                C   C31=-SINP*COSL
                C   C32=-SINP*SINL
                C   C33=COSP
                C
0117                   GU=COSP*COSL*GX + COSP*SINL*GY + SINP*GZ
0118                   GE=-SINL*GX + COSL*GY
0119                   GN=-SINP*COSL*GX - SINP*SINL*GY + COSP*GZ
                C
                C   STORE THE OBSERVATION DISTURBANCES
                C
0120              200 DELGU(MJ)=GU
0121                  DELGE(MJ)=GE
0122                  DELGN(MJ)=GN
0123              250 CONTINUE
                C
                C   WRITE THE THREE OBSERVATION ARRAYS ONTO THE SEQUENTIAL FILE
                C
0124                   WRITE(3) (DELGU(I),I=1,MOBS)
0125                   WRITE(3) (DELGE(I),I=1,MOBS)
0126                   WRITE(3) (DELGN(I),I=1,MOBS)
0127              300 CONTINUE
0128              500 STOP
0129                  END
```

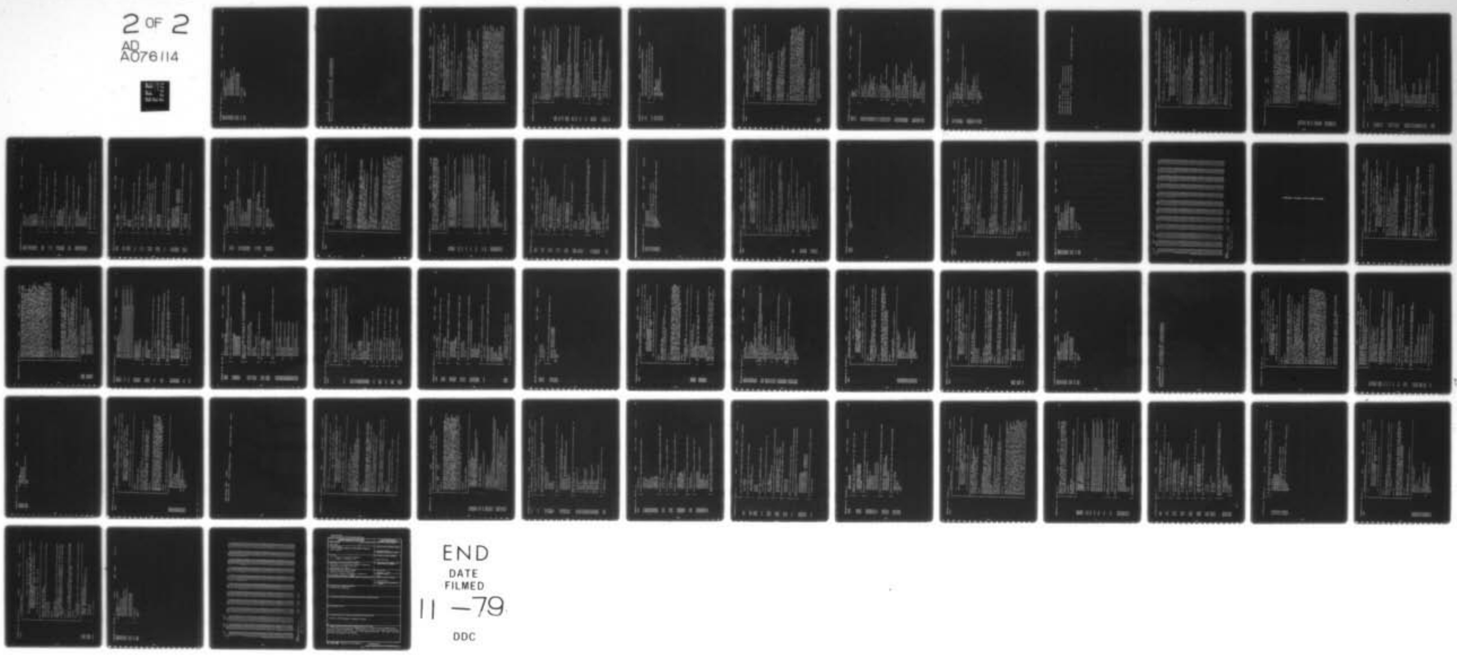
```
FORTRAN IV G1   RELEASE 2.0        CHEBY              DATE = 79206        20/33/12            PA
0001              SUBROUTINE CHEBY(X,N,TA)
            C-----------------------------------------------------------------------
            C
            C
            C     BY                  JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
            C           DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
            C           SCHOOL OF ENGINEERING
            C           VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
            C           BLACKSBURG, VIRGINIA    24061
            C
            C     DATE OF LAST MODIFICATION -- APRIL 1,1979
            C
            C-----------------------------------------------------------------------
            C
            C  INPUTS
            C
            C  FROM SUBROUTINE CALL
            C  X--THE NORMALIZED COORDINATE WHERE THE FUNCTION IS TO BE EVALUATED
            C  N--ORDER OF THE BASIS FUNCTION DESIRED
            C
            C  PROCESS
            C
            C  THIS SUBROUTINE EVALUATES THE BASIS FUNCTION AT THE NORMALIZED
            C  COORDINATE VALUE X, FOR EACH ORDER OF THE FUNCTION SPECIFIED (N).
            C  THE FUNCTION VALUES ARE RETURNED IN VECTOR TA.
            C
            C  OUTPUTS
            C
            C  TO SUBROUTINE RETURN
            C  TA--FUNCTION VALUES OF ORDER N EVALUATED AT X
            C
            C-----------------------------------------------------------------------
0002              IMPLICIT REAL*8 (A-H,O-Z)
0003              DIMENSION  TA(7)
            C
0004              IF (N .GT. 0 .AND. N .LT. 7) GOTO 10
0005              WRITE(6,5) N
0006            5 FORMAT(24H0ILLEGAL NORDER, NORDER=,I6)
0007              STOP
0008           10 CONTINUE
            C
            C  TRANSFORM THE COORDINATE RANGE (0,1) TO (-1,1)
            C
0009              XBAR=2.D0*X-1.D0
0010              TA(1)=1.D0
0011              TA(2)=XBAR
0012              NP1=N+1
0013              DO 20 I=3,NP1
0014           20 TA(I)=2.D0*XBAR*TA(I-1)-TA(I-2)
0015              RETURN
0016              END
```

```
      SUBROUTINE PTMASS
C-----------------------------------------------------------------------
C
C
C     BY                JOHN L. JUNKINS AND JOHN SAUNDERS
C
C     MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C            DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C            SCHOOL OF ENGINEERING
C            VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C            BLACKSBURG, VIRGINIA    24061
C
C     DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C-----------------------------------------------------------------------
C
C  FOR P,  SOME POINT IN SPACE
C  DISTURBANCE IS TO BE EVALUATED
C  XI,YI,ZI--EARTH-FIXED RECTANGULAR COORDINATES OF THE ITH POINT MASS
C  POSITS(I,1)=XI
C  POSITS(I,2)=YI
C  POSITS(I,3)=ZI
C
C  INPUTS
C
C  FROM COMMON XYZ
C  X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES OF POINT AT WHICH GRAVITY
C  FROM COMMON MASPOS
C  PMVALS--PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL CONSTANT AND THE
C     1080 POINT MASSES (-1.E19, 0., OR +1.E19)
C  POSITS--PRECOMPUTED EARTH-FIXED X,Y,Z COORDINATES OF THE 1080 POINT
C     MASSES
C
C  PROCESS
C
C  GIVEN THE RECTANGULAR COORDINATES, X, Y, Z, OF A POINT, THIS ROUTINE
C  RETURNS THE COMPONENTS OF THE GRAVITY DISTURBANCE, DELGX, DELGY, AND
C  DELGZ, USING THE POINT MASSES OF MASS MODEL 310.
C
C  OUTPUTS
C
C  TO COMMON XYZ
C  DELGX,DELGY,DELGZ--EARTH-FIXED RECTANGULAR COMPONENTS OF THE GRAVITY
C     DISTURBANCE
C
C-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /XYZ/X,Y,Z,DELGX,DELGY,DELGZ
      COMMON  /MASPOS/PMVALS(1080),POSITS(1080,3)
C
      DELGX=0.D0
      DELGY=0.D0
      DELGZ=0.D0
C
      DO 20 I=1,1080
```

AD-A076 114 VIRGINIA POLYTECHNIC INST AND STATE UNIV BLACKSBURG --ETC F/G 8/5
FINITE ELEMENT MODELS OF THE EARTH'S GRAVITY FIELD.(U)
OCT 79 R C ENGELS , J L JUNKINS DAAK70-78-C-0072

UNCLASSIFIED ETL-0198 NL

2 OF 2
AD
A076114



END
DATE
FILMED
11 -79
DDC

```
      IF (PMVALS(I)) 10,20,10
   10 CONTINUE
      DX=POSITS(I,1)-X
      DY=POSITS(I,2)-Y
      DZ=POSITS(I,3)-Z
      DISTSQ=DX*DX+DY*DY+DZ*DZ
      DIST=DSQRT(DISTSQ)
      TEMP=PMVALS(I)/DIST/DISTSQ
C
      DELGX=DELGX+(DX*TEMP)
      DELGY=DELGY+(DY*TEMP)
      DELGZ=DELGZ+(DZ*TEMP)
C
   20 CONTINUE
C
      RETURN
      END
```

```
   3         20
   4          4            4              64
0.300000000000D+00   0.100000000000D+01   0.100000000000D+01
0.0                  0.750000000000D+02  -0.300000000000D+02
0.300000000000D+00   0.780000000000D+02  -0.290000000000D+02
```

```
      PROGRAM FINEG(OUTPUT,TAPE6=OUTPUT,TAPE1,TAPE3)                    *
C-----------------------------------------------------------------------
C
C    SECTION III
C     LOCAL GRAVITY MODEL -- COEFFICIENT DETERMINATION PROGRAM
C
C     BY                 JOHN L. JUNKINS AND JOHN SAUNDERS
C
C     MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C             DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C             SCHOOL OF ENGINEERING
C             VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C             BLACKSBURG, VIRGINIA   24061
C
C     DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C-----------------------------------------------------------------------
C
C   FOR P,  SOME POINT IN SPACE
C   X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES
C   H,ALAM,APHI--ELLIPSOIDAL COORDINATES
C   KORDER--ORDER OF THE BASIS FUNCTIONS DESIRED
C
C   INPUTS
C   FROM DISK
C   FILE 3
C   HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
C   HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
C   NH,NLAM,NPHI--NUMBER OF CELLS IN H,LAMDA,PHI DIRECTION
C   NORDER--ORDER OF POLYNOMIALS DESIRED
C   NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
C   A--BASIS FUNCTION MATRIX FOR LEAST SQUARES FIT
C   DELGU,DELGE,DELGN--GRAVITY DISTURBANCE COMPONENTS IN H,LAMDA,PHI
C
C   PROCESS
C
C   THIS PROGRAM ACCEPTS FROM DISK , INPUT DESCRIBING THE LIMITS OF THE
C   REGION TO BE MODELED, THE ORDER OF THE BASIS FUNCTIONS TO BE USED IN
C   THE MODELING, A LEAST SQUARES MATRIX MADE UP OF BASIS FUNCTION
C   PRODUCTS AND SETS OF THREE VECTORS OF GRAVITY ANOMALY OBSERVATIONS
C   FOR EACH CELL THAT MAKE UP THE REGION.  THE REGION DESCRIPTION
C   INFORMATION IS WRITTEN INTO THE FIRST RECORD OF A RANDOM ACCESS FILE
C   (FILE 1).  THE LEAST SQUARES MATRIX IS THEN REDUCED TO AN UPPER TRI-
C   ANGULAR FORM BY SUBROUTINE ALSQ.  THE OBSERVATION VECTORS ARE THEN
C   READ IN ONE AT A TIME AND THE SECOND SECTION OF ALSQ (ALSQ1), DE-
C   TERMINES THE COEFFICIENTS WHICH ARE NECESSARY TO FIT THE BASIS
C   FUNCTIONS TO THE OBSERVED DATA.  THIS FITTING PROCESS IS CONTINUED
C   FOR EACH CELL.  ALL THREE SETS OF COEFFICIENTS WHICH DESCRIBE A
C   GIVEN CELL ARE THEN WRITTEN INTO THE NEXT RECORD OF THE RANDOM
C   ACCESS FILE.  SINCE THE COEFFICIENTS ARE DETERMINED SEPARATELY, THE
C   FINAL MODELING PROCESS, PERFORMED BY PROGRAM FINITE IN SECTION III,
C   MUST USE THREE SETS OF COEFFICIENTS TO DETERMINE THE THREE GRAVITY
C   ANOMALY COMPONENTS.
C
```

```
FORTRAN IV G1  RELEASE 2.0            MAIN                  DATE = 79206        20/33/39

                 C  OUTPUT
                 C
                 C  TO DISK
                 C  FILE 1
                 C  HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
                 C  HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
                 C  HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
                 C  NH,NLAM,NPHI--NUMBER OF CELLS IN H,LAMDA,PHI DIRECTION
                 C  NORDER--ORDER OF POLYNOMIALS DESIRED
                 C  NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
                 C  CO,CE,CN--MODELING COEFFICIENTS FOR THE H,LAMDA,PHI COMPONENTS OF
                 C   THE GRAVITY ANOMALY
                 C
                 C---------------------------------------------------------------------------
0001                  IMPLICIT REAL*8 (A-H,O-Z)
0002                  COMMON  /FLDATA/HMIN,HMAX,HCELL,ALMIN,ALMAX,ALCELL,APMIN,APMAX,
                     +APCELL,NH,NLAM,NPHI,NORDER,NC
0003                  DIMENSION A(344,85),DELG(343)
0004                  DIMENSION CO(84),CE(84),CN(84)
0005                  DATA  MAXOBI/344/
0006                  DEFINE FILE 1(251,2016,U,IPOINT)
                 C
0007                6 FORMAT(1X,5E12.4)
0008                7 FORMAT(1H0,10X,18HALSQ CALLED, TIME=,F10.4)
0009                8 FORMAT(1H0,10X,18HCOEFFICIENTS FOR,I5,25H CELLS COMPUTED, DELTAT=
                     +,F10.4,14H AVERAGE TIME=,F10.4)
                 C
0010                  CALL TIMEON
0011                  READ(3) HMIN,HMAX,HCELL,ALMIN,ALMAX,ALCELL,APMIN,APMAX,APCELL,NH,
                     +NLAM,NPHI,NORDER,NC,MOBS,NCELLS
0012                  WRITE(1'1) HMIN,HMAX,HCELL,ALMIN,ALMAX,ALCELL,APMIN,APMAX,APCELL,
                     +NH,NLAM,NPHI,NORDER,NC
                 C
0013                  KNCELL=NCELLS
                 C
                 C  READ THE LEAST SQUARES MATRIX FROM THE SEQUENTIAL FILE
                 C
0014                  READ(3) ((A(I,J),J=1,NC),I=1,MOBS)
                 C
                 C  REDUCE THE LEAST SQUARES MATRIX TO UPPER TRIANGULAR FORM
                 C
0015                  CALL ALSQ(A,DELG,CO,SUMSQ,MOBS,NC,MAXOBI)
0016                  CALL TIMECK(ICP)
0017                  TIME=ICP/100.D0
0018                  WRITE(6,7) TIME
                 C
                 C  FOR EACH CELL, READ ONE SET OF GRAVITY OBSERVATIONS AND
                 C  COMPUTE THE COEFFICIENT ARRAY FOR EACH COMPONENT OF GRAVITY
                 C
0019                  CALL TIMEON
0020                  DO 40 ID=1,NCELLS
0021                  READ(3) (DELG(I),I=1,MOBS)
0022                  CALL ALSQ1(A,DELG,CO,SUMSQ,MOBS,NC,MAXOBI)
                 C
0023                  READ(3) (DELG(I),I=1,MOBS)
```

```
0024          CALL ALSQ1(A,DELG,CE,SUMSQ,MOBS,NC,MAXOB1)
        C
0025          READ(3) (DELG(I),I=1,MOBS)
0026          CALL ALSQ1(A,DELG,CN,SUMSQ,MOBS,NC,MAXOB1)
        C
        C  STORE EACH COEFFICIENT ARRAY IN A RANDOM ACCESS FILE
        C
0027          WRITE(1'IPOINT) (CO(IC),CE(IC),CN(IC),IC=1,NC)
        C
0028       40 CONTINUE
0029          CALL TIMECK(ICP)
0030          TIME=ICP/100.D0
0031          TAV=TIME/KNCELL
0032          WRITE(6,8) NCELLS,TIME,TAV
0033       50 STOP
0034          END
```

```
0001        SUBROUTINE ALSQ(A,Y,B,R2,NN,MM,NA)
      C-----------------------------------------------------------------------
      C
      C     BY                     JOHN L. JUNKINS AND JOHN SAUNDERS
      C
      C     MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
      C           DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
      C           SCHOOL OF ENGINEERING
      C           VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
      C           BLACKSBURG, VIRGINIA   24061
      C
      C     DATE OF LAST MODIFICATION -- APRIL 1,1979
      C
      C-----------------------------------------------------------------------
      C
      C  FOR P,   SOME POINT IN SPACE
      C  X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES
      C  H,ALAM,APHI--ELLIPSOIDAL COORDINATES
      C  NORDER--ORDER OF THE BASIS FUNCTIONS DESIRED
      C
      C  INPUTS
      C
      C  FROM SUBROUTINE CALL
      C  A--BASIS FUNCTION MATRIX FOR LEAST SQUARES FIT
      C  Y--OBSERVATION VECTOR TO BE FIT
      C  N--NUMBER OF ROWS IN THE LEAST SQUARES MATRIX
      C  M--NUMBER OF COLUMNS IN THE LEAST SQUARES MATRIX
      C  NA--NUMBER OF STORAGE ELEMENTS RESERVED FOR MATRIX A
      C
      C
      C  PROCESS
      C
      C  THIS SUBROUTINE IS USED BY PROGRAM FINEG FOR TWO OPERATIONS.  THE
      C  FIRST IS TO REDUCE THE LEAST SQUARES MATRIX TO AN UPPER TRIANGULAR
      C  FORM.  THIS IS PERFORMED BY A CALL TO ENTRY ALSQ.  SINCE THE LEAST
      C  SQUARES MATRIX, A, IS THE SAME FOR EACH CELL THIS REDUCTION IS ONLY
      C  NECESSARY ONCE.  THE SECOND OPERATION IS INVOKED BY A CALL TO ENTRY
      C  ALSQ1.  THIS SECTION USES THE REDUCED A MATRIX AND THE SUPPLIED
      C  VECTOR Y WHICH CONTAINS THE OBSERVATIONS TO WHICH THE BASIS FUNCTIONS
      C  ARE TO BE FIT, TO CALCULATE THE COEFFICIENTS THAT ARE NECESSARY
      C  TO PRODUCE A LEAST SQUARES FIT.  THESE COEFFICIENTS AND THE RESID-
      C  UAL SUM OF THE SQUARES, WHICH GIVES AN INDICATIONS OF THE ACCURACY
      C  OF THE FIT, ARE RETURNED TO FINEG.
      C
      C  OUTPUT
      C
      C  TO SUBROUTINE RETURN
      C  A--THE (M+1)TH COLUMN CONTAINS THE APPROXIMATING VECTOR AB
      C  B--COEFFICIENTS OF THE FIT
      C  R2--RESIDUAL SUM OF THE SQUARES
      C
      C-----------------------------------------------------------------------
0002        IMPLICIT REAL*8 (A-H,O-Z)
0003        DIMENSION A(NA,1),Y(1),B(1)
0004        N=NN
```

```
      N1=N+1
      M=MM
      M1=M+1
      MM1=M-1
C
C     REDUCE THE LEAST SQUARES MATRIX TO UPPER TRIANGULAR FORM
C
      DO 60 L=1,M
      SS=0.D0
      DO 10 I=L,N
   10 SS=SS+A(I,L)**2
      S2=SS
      S=DSQRT(S2)
      IF(A(L,L).LT.0.) S=-S
      D=S2 + S*A(L,L)
      A(L,L)=A(L,L) + S
      IF(L.EQ.M) GO TO 50
      L1=L+1
      DO 30 J=L1,M
      PP=0.D0
      DO 20 I=L,N
   20 PP=PP + A(I,L)*A(I,J)
   30 A(N1,J)=PP/D
      DO 40 J=L1,M
      DO 40 I=L,N
   40 A(I,J)=A(I,J) - A(I,L)*A(N1,J)
   50 A(N1,L)=-S
   60 CONTINUE
      RETURN
C
C     REDUCE THE VECTOR Y
C
      ENTRY ALSQ1(A,Y,B,R2,NN,MM,NA)
   70 DO 80 I=1,N
   80 A(I,M1)=Y(I)
      DO 100 L=1,M
      PP=0.D0
      DO 90 I=L,N
   90 PP = PP +A(I,L)*A(I,M1)
      D=PP/(-A(L,L)*A(N1,L))
      DO 100 I=L,N
  100 A(I,M1)= A(I,M1) - D*A(I,L)
C
C     CALCULATE THE COEFFICIENT VECTOR B
C
      B(M)=A(M,M1)/A(N1,M)
      IF(M.EQ.1) GO TO 130
      DO 120 LL=1,MM1
      L=M-LL
      L1=L+1
      PP=A(L,M1)
      DO 110 I=L1,M
  110 PP=PP-A(L,I)*B(I)
  120 B(L)=PP/A(N1,L)
C
```

```
                C CALCULATE R2
                C
0050              130 SS=0.D0
0051                  MP1=M+1
0052                  DO 140 I=MP1,N
0053                  SS=SS+A(I,M1)**2
0054              140 A(I,M1)=0.D0
0055                  R2=SS
0056                  WRITE(6,901) R2
0057              901 FORMAT(1H0,10X,24HRESIDUAL SUM OF SQUARES=,E20.12)
                C
                C     PERFORM THE BACK CALCULATIONS
                C
0058                  DO 170 LL=1,M
0059                  L=M-LL+1
0060                  PP=0.D0
0061                  DO 150 I=L,N
0062              150 PP=PP+A(I,L)*A(I,M1)
0063                  D=PP/(-A(L,L)*A(N1,L))
0064                  DO 160 I=L,N
0065              160 A(I,M1)=A(I,M1)-D*A(I,L)
0066              170 CONTINUE
0067                  RETURN
0068                  END
```

```
ALSO CALLED, TIME=     0.4100
RESIDUAL SUM OF SQUARES=  0.39550300606590-08
RESIDUAL SUM OF SQUARES=  0.17114484815 1D-08
RESIDUAL SUM OF SQUARES=  0.17117187342 7D-08
COEFFICIENTS FOR     1 CELLS COMPUTED, DELTAT=     0.1200 AVERAGE TIME=     0.1200
```

```
      PROGRAM FINTES(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1,TAPE2) *
C------------------------------------------------------------------------
C
C   SECTION III
C     LOCAL GRAVITY MODEL -- ERROR ANALYSIS  AND TIME COMPARISON
C
C     BY                   JOHN L. JUNKINS AND JOHN SAUNDERS
C
C     MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C            DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C            SCHOOL OF ENGINEERING
C            VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C            BLACKSBURG, VIRGINIA   24061
C
C     DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C------------------------------------------------------------------------
C
C   FOR P,  SOME POINT IN SPACE
C   X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES
C   H,ALAM,APHI--ELLIPSOIDAL COORDINATES
C   RN--EAST-WEST RADIUS OF CURVATURE
C   NORDER--ORDER OF THE BASIS FUNCTIONS DESIRED
C   H--HEIGHT ABOVE REF. ELLIPSOID ALONG NORMAL TO P
C   ALAM--ANGLE LAMBDA, GEODETIC/GEOCENTRIC LONGITUDE (RADIANS) OF P
C   APHI--ANGLE PHI, GEODETIC LATITUDE (RADIANS) OF P
C
C   INPUTS
C
C   FROM DISK
C   FILE 1
C   HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
C   HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
C   NH,NLAM,NPHI--NUMBER OF CELLS IN H,LAMDA,PHI DIRECTION
C   NORDER--ORDER OF POLYNOMIALS DESIRED
C   NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
C   FILE 2
C   PMVALS--PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL CONSTANT AND THE
C      1080 POINT MASSES (-1.E19, 0., OR +1.E19)
C   POSITS--PRECOMPUTED EARTH-FIXED X,Y,Z COORDINATES OF THE 1080 POINT
C      MASSES
C   FROM CARDS
C   FORMAT 1
C   ISTEPH,ISTEPL,ISTEPP--TEST GRID PATTERN
C
C   THE CARD INPUT FOR THIS SAMPLE RUN IS AS FOLLOWS:
C    4    4    4
C
C   FORMAT 2
C   HMIN,ALMIN,APMIN--MINUMUM BOUNDS OF TEST FIELD DESIRED
C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF TEST FIELD DESIRED
C
C   THE CARD INPUT FOR THIS SAMPLE RUN IS AS FOLLOWS:
C  1.        75.1        -29.9
```

```
           C270001.    75.9       -29.1
           C
           C   PROCESS
           C
           C   THIS PROGRAM ACCEPTS CARD INPUT WHICH SPECIFIES THE MINIMUM AND
           C   MAXIMUM LIMITS OF A TEST REGION WHICH OVERLAPS OR IS CONTAINED IN THE
           C   REGION ESTABLISHED AND MODELLED IN SECTION II.  THE ENTIRE REGION
           C   CONSTITUTES A SINGLE CELL WHICH IS AGAIN DIVIDED BY A REGULAR GRID
           C   PATTERN.  AT EACH GRID OR OBSERVATION POINT THE ACTUAL GRAVITY
           C   DISTURBANCES ARE CALCULATED USING MASS MODEL 310 (SUBROUTINE PTMASS)
           C   AND THE MODELLED GRAVITY DISTURBANCES ARE CALCULATED BY USING THE
           C   MODELLING EQUATION WHOSES COEFFICIENTS WERE DETERMINED IN SECTION
           C   II AND STORED ON A RANDOM ACCESS FILE (FILE 1).AN ERROR ANALYSIS IS
           C   PERFORMED ON EACH TEST POINT AND THE RESULTS ARE OUTPUT.  IN ADDI-
           C   TION TO THE POINT BY POINT ANALYSIS, THE MEANS, STANDARD DIVIATIONS,
           C   AND MAXIMUM ABSOLUTE ERRORS ARE OUTPUT ALONG WITH THE TOTAL
           C   EXECUTION TIMES OF THE TWO METHODS.
           C
           C   OUTPUTS
           C
           C   ERROR ANALYSIS REPORT
           C
           C----------------------------------------------------------------------
0001             IMPLICIT REAL*8 (A-H,O-Z)
0002             COMMON  /HLP/H,ALAM,APHI,GU,GE,GN
0003             COMMON  /XYZ/X,Y,Z,GX,GY,GZ
0004             COMMON/IMARK/IFLAG,ISTTIN
0005             COMMON /FLDATA/REA(9),INT(5)
0006             COMMON  /MASPOS/PMVALS(1080),POSITS(1080,3)
           C
0007             DATA A/6378160.D0/
0008             DATA B/6356774.5040D0/
           C
0009             DEFINE FILE 1(251,2016,U,IPOINT)
           C
0010           1 FORMAT(16I5)
0011           2 FORMAT(F10.0,2F10.2)
0012           3 FORMAT(1H1)
0013           4 FORMAT(1X,12I10)
0014           5 FORMAT(1X,F10.0,2F10.2)
0015           6 FORMAT(1H0,4X,1HH,8X,6HLAMBDA,5X,3HPHI,7X,5HERRGU,5X,5HERRGE,5X
                +5HERRGN,5X,4HPMGU,6X,4HPMGE,6X,4HPMGN,6X,4HFIGU,6X,4HFIGE,6X,
                +4HFIGN)
0016           7 FORMAT(4X,4H(KM),1X,2(5X,5H(DEG)),2X,9(3X,7H(MGALS)))
0017           8 FORMAT(2X,9(1H-),11(2X,8(1H-)))
0018           9 FORMAT(1H0,10X,22HFINITE TIME/EXECUTION=,F9.6)
0019          10 FORMAT(1H0,10X,22HPTMASS TIME/EXECUTION=,F9.6)
0020          11 FORMAT(1X,-3PF10.3,2(0PF10.4),9(1X,5PF9.4))
0021          12 FORMAT(///,1X,5HMEANS,25X,3(1X,5PF9.4))
0022          13 FORMAT(1X,19HSTANDARD DEVIATIONS,11X,3(1X,5PF9.4))
0023          14 FORMAT(1X,26HABS. VALUES OF MAX. ERRORS,4X,3(1X,5PF9.4))
0024          15 FORMAT(46H0*****   NEGATIVE OR ZERO NO. OF STEPS    *****,
           C
           C   READ IN THE FIELD DATA FROM THE FIRST RECORD OF THE RANDOM ACCESS
           C   FILE (FILE 1).
```

```
              C
0025                READ(1'1) REA,INT
              C
              C   READ FILE 2 WHICH CONTAINS THE MASS VALUES AND THEIR X,Y,Z COORDINATE
              C   SET UP CONVERSION FACTORS AND INITIALIZATION
              C
0026                READ(2) PMVALS,POSITS
0027                PI=DARCOS(-1.D0)
0028                DEGRAD=PI/180.D0
0029                A2=A*A
0030                B2=B*B
0031                IFLAG=0
0032                ISTTIN=0
              C
              C   READ IN THE TEST CELL GRID PATTERN AND THE FINITE ELEMENT FIELD
              C   BOUNDS
              C
0033             20 READ(5,1,END=40) ISTEPH,ISTEPL,ISTEPP
0034                READ(5,2,END=40) HMIN,ALMIN,APMIN,HMAX,ALMAX,APMAX
0035                WRITE(6,3)
0036                WRITE(6,4) ISTEPH,ISTEPL,ISTEPP
0037                WRITE(6,5) HMIN,ALMIN,APMIN,HMAX,ALMAX,APMAX
0038                IHM1=ISTEPH-1
0039                ILM1=ISTEPL-1
0040                IPM1=ISTEPP-1
              C
              C   VERIFY THAT THE TEST CELL GRID PATTERN IS VALID AND CALCULATE THE
              C   GRID INCREMENT SIZE
              C
0041                IF (IHM1)  21,22,23
0042             21 WRITE(6,15)
0043                GOTO 40
0044             22 DH=0.D0
0045                GOTO 24
0046             23 RHM1=IHM1
0047                DH=(HMAX-HMIN)/RHM1
0048             24 IF (ILM1)  21,25,26
0049             25 DLAM=0.D0
0050                GOTO 27
0051             26 RLM1=ILM1
0052                DLAM=(ALMAX-ALMIN)/RLM1
0053             27 IF (IPM1)  21,28,29
0054             28 DPHI=0.D0
0055                GOTO 30
0056             29 RPM1=IPM1
0057                DPHI=(APMAX-APMIN)/RPM1
              C
              C   SET UP THE ERROR ANALYSIS REPORT HEADINGS
              C
0058             30 WRITE(6,6)
0059                WRITE(6,7)
0060                WRITE(6,8)
              C
              C   INITIALIZE ALL TIME AND ERROR ANALYSIS VARIABLES
              C
```

```
0061                FTIME=0.D0
0062                PTIME=0.D0
0063                EGU=0.D0
0064                EGE=0.D0
0065                EGN=0.D0
0066                SDEGU=0.D0
0067                SDEGE=0.D0
0068                SDEGN=0.D0
0069                EGUMAX=0.D0
0070                EGEMAX=0.D0
0071                EGNMAX=0.D0
              C
              C   DETERMINE THE 3 GRAVITY DISTURBANCES AT EACH TEST GRID POINT
              C
0072                H=HMIN-DH
0073                DO 35  I=1,ISTEPH
              C
              C   INCREMENT THE H COORDINATE 1 GRID INCREMENT SIZE
              C
0074                H=H+DH
0075                AL=ALMIN-DLAM
0076                DO 35  J=1,ISTEPL
              C
              C   INCREMENT THE LAMDA COORIDINATE 1 GRID INCREMENT SIZE
              C
0077                AL=AL+DLAM
0078                ALAM=AL*DEGRAD
0079                COSL=DCOS(ALAM)
0080                SINL=DSIN(ALAM)
0081                AP=APMIN-DPHI
0082                DO 35  K=1,ISTEPP
              C
              C   INCREMENT THE PHI COORDINATE 1 GRID INCREMENT SIZE
              C
0083                AP=AP+DPHI
0084                APHI=AP*DEGRAD
              C
              C   TRANSFORM THE ELLIPSOIDAL COORDINATES INTO X,Y,Z
              C
0085                COSP=DCOS(APHI)
0086                SINP=DSIN(APHI)
0087                RN=DSQRT(A2*COSP*COSP + B2*SINP*SINP)
0088                ZN=B2/RN
0089                RN=A2/RN
0090                Y=(RN+H)*COSP
0091                X=Y*COSL
0092                Y=Y*SINL
0093                Z=(ZN+H)*SINP
              C
              C   EVALUATE THE GRAVITY DISTURBANCES USING THE MODELING EQUATION
              C   WHOSE COEFFICIENTS WHERE DETERMINED IN SECTION II
              C
              C
              C   START THE TIMER TO DETERMINE THE EXECUTION TIME OF THE EQUATION
              C
```

```
0094               CALL TIMEON
0095               CALL FINITE
             C
             C   STOP THE TIMER AND CONVERT THE TIME TO SECONDS
             C
0096               CALL TIMECK(ICP)
0097               FTIME=FTIME + ICP/100.D0
             C
0098               GUF=GU
0099               GEF=GE
0100               GNF=GN
             C
             C   START TIMER TO DETERMINE THE EXECUTION TIME
             C
0101               CALL TIMEON
             C
             C   EVALUATE THE GRAVITY DISTURBANCES USING MASS MODEL 310
             C
0102               CALL PTMASS
0103               CALL TIMECK(ICP)
0104               PTIME=PTIME + ICP/100.D0
             C
             C   TRANSFORM THE GRAVITY DISTURBANCES TO ELLIPSOIDAL COORDINATES
             C
0105               GU=COSP*COSL*GX + COSP*SINL*GY + SINP*GZ
0106               GE=-SINL*GX + COSL*GY
0107               GN=-SINP*COSL*GX - SINP*SINL*GY + COSP*GZ
             C
             C   DETERMINE THE ERROR VALUE BETWEEN THE TWO METHODS
             C
0108               ERRGU=GU-GUF
0109               ERRGE=GE-GEF
0110               ERRGN=GN-GNF
             C
             C   OUTPUT THE ERROR ANALYSIS FOR THIS GRID POINT
             C
0111               WRITE(6,11) H,AL,AP,ERRGU,ERRGE,ERRGN,GU,GE,GN,GUF,GEF,GNF
             C
             C   DETERMINE THE MAXIMUM ABSOLUTE ERROR VALUE FOR EACH COORDINATE
             C   DISTURBANCE
             C
0112               AERRGU=DABS(ERRGU)
0113               AERRGE=DABS(ERRGE)
0114               AERRGN=DABS(ERRGN)
0115               IF(AERRGU.GT.EGUMAX)  EGUMAX=AERRGU
0116               IF(AERRGE.GT.EGEMAX)  EGEMAX=AERRGE
0117               IF(AERRGN.GT.EGNMAX)  EGNMAX=AERRGN
             C
             C   CALCULATE THE SUM OF THE ERROR FOR EACH COORDINATE DISTURBANCE
             C
0118               EGU=EGU+ERRGU
0119               EGE=EGE+ERRGE
0120               EGN=EGN+ERRGN
             C
             C   CALCULATE THE SUM OF THE SQUARES OF THE ERRORS FOR EACH
```

```
FORTRAN IV G1  RELEASE 2.0            MAIN              DATE = 79206        20/34/03        PA

                C  COORDINATE DISTURBANCE
                C
 0121              SDEGU=SDEGU+ERRGU*ERRGU
 0122              SDEGE=SDEGE+ERRGE*ERRGE
 0123              SDEGN=SDEGN+ERRGN*ERRGN
 0124           35 CONTINUE
                C
                C  CALCULATE THE MEAN AND STANDARD DEVIATION OF THE ERRORS
                C
 0125              IPTS=ISTEPH*ISTEPL*ISTEPP
 0126              RIPTS=IPTS
 0127              RIPTM1=RIPTS-1.D0
 0128              EGU=EGU/RIPTS
 0129              EGE=EGE/RIPTS
 0130              EGN=EGN/RIPTS
 0131              SDEGU=DSQRT(SDEGU/RIPTM1)
 0132              SDEGE=DSQRT(SDEGE/RIPTM1)
 0133              SDEGN=DSQRT(SDEGN/RIPTM1)
                C
                C  OUTPUT THE MEANS, STANDARD DEVIATION AND MAXIMUM ABSOLUTE ERROR
                C
 0134              WRITE(6,12) EGU,EGE,EGN
 0135              WRITE(6,13) SDEGU,SDEGE,SDEGN
 0136              WRITE(6,14) EGUMAX,EGEMAX,EGNMAX
 0137              FTIME=FTIME/RIPTS
 0138              PTIME=PTIME/RIPTS
                C
                C  OUTPUT THE AVERAGE EXECUTION TIME PER POINT FOR EACH METHOD
                C
 0139              WRITE(6,9) FTIME
 0140              WRITE(6,10) PTIME
 0141              GOTO 20
 0142           40 CONTINUE
 0143              STOP
 0144              END
```

```
0001          SUBROUTINE FINITE
        C-----------------------------------------------------------------------
        C
        C      BY                  JOHN L. JUNKINS AND JOHN SAUNDERS
        C
        C      MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
        C            DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
        C            SCHOOL OF ENGINEERING
        C            VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
        C            BLACKSBURG, VIRGINIA   24061
        C
        C      DATE OF LAST MODIFICATION -- APRIL 1,1979
        C
        C-----------------------------------------------------------------------
        C
        C   FOR P,  SOME POINT IN SPACE
        C   X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES
        C   H,ALAM,APHI--ELLIPSOIDAL COORDINATES
        C   RN--EAST-WEST RADIUS OF CURVATURE
        C   NORDER--ORDER OF THE BASIS FUNCTIONS DESIRED
        C
        C   INPUTS
        C
        C   FROM COMMON FEDATA
        C   HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
        C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
        C   HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
        C   NH,NLAM,NPHI--NUMBER OF CELLS IN H,LAMDA,PHI DIRECTION
        C   NORDER--ORDER OF POLYNOMIALS DESIRED
        C   NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
        C   FROM DISK
        C   FILE 1
        C   CU,CE,CN--MODELING COEFFICIENTS FOR THE H,LAMDA,PHI COMPONENTS OF
        C     THE GRAVITY ANOMALY
        C   FROM COMMON HLP
        C   H--HEIGHT ABOVE REF. ELLIPSOID ALONG NORMAL TO P
        C   ALAM--ANGLE LAMBDA, GEODETIC/GEOCENTRIC LONGITUDE (RADIANS) OF P
        C   APHI--ANGLE PHI, GEODETIC LATITUDE (RADIANS) OF P
        C
        C   PROCESS
        C
        C   THIS PROGRAM ACCEPTS INPUT OF ELLIPSOIDAL COORDINATES (H,LAMDA,PHI)
        C   FROM THE MAIN PROGRAM FINITES AND EVALUATES THE THREE GRAVITY
        C   ANOMALIES AT THIS POINT.  FIRST THE POINT IS TESTED TO SEE IF IT
        C   LIES WITHIN THE REGION MODELED IN SECTION II.  IF NOT, THE GRAVITY
        C   DISTURBANCES ARE EVALUATED BY SUBROUTINE PTMASS WHICH USES MASS
        C   MODEL 310, AND A MESSAGE IS PRINTED INDICATING THIS FACT.  THE CELL
        C   (IF THERE IS MORE THAN ONE CELL THAT MAKES UP THE REGION) WHICH
        C   CONTAINS THIS POINT IS DETERMINED AND THE CORE MEMORY IS CHECKED TO
        C   SEE IF THE MODELING COEFFICIENTS FOR THIS CELL ARE AVAILABLE IN CORE.
        C   IF NOT, THE COEFFICIENTS ARE READ IN FROM FILE 1 WHICH WAS CREATED
        C   IN SECTION II.  WHEN THE COEFFICIENTS ARE AVAILABLE THE COORDINATES
        C   ARE NORMALIZED AND THE BASIS FUNCTIONS ARE EVALUATED AT THIS POINT
        C   FOR EACH ORDER OF THE POLYNOMIALS UP TO THE SPECIFIED ORDER.  EACH
        C   COEFFICIENT IS THEN MULTIPLIED BY THREE BASIS FUNCTIONS OF PREDETER-
```

```
      C    MINED ORDERS.  THIS GIVES THE CONTRIBUTION OF EACH COEFFICIENT TO
      C    EACH OF THE THREE GRAVITY ANOMALIES.  WHEN THE THREE CONTRIBUTIONS
      C    OF ALL THE COEFFICIENTS ARE SUMMED SEPARATELY, THE RESULTS ARE THE
      C    H,LAMDA,PHI COMPONENTS OF THE GRAVITY ANOMALY AT THE GIVEN POINT.
      C    THESE COMPONENTS ARE RETURNED TO THE MAIN PROGRAM FINTES.
      C
      C    OUTPUTS
      C
      C    TO COMMON HLP
      C    GU,GE,GN--ELLIPSOIDAL COMPONENTS OF THE GRAVITY DISTURBANCE
      C
      C---------------------------------------------------------------------------
0002         IMPLICIT REAL*8 (A-H,O-Z)
0003         COMMON  /HLP/H,ALAM,APHI,GU,GE,GN
0004         COMMON  /XYZ/X,Y,Z,GX,GY,GZ
0005         COMMON/IMARK/IFLAG,ISITIN
0006         COMMON /FLDATA/HMIN,HMAX,HCELL,ALMIN,ALMAX,ALCELL,APMIN,APMAX,
            +APCELL,NH,NLAM,NPHI,NORDER,NC
      C
0007         DIMENSION IX(7),IY(7),IZ(7),CU(84),CE(84),CN(84)
0008         DIMENSION  IX(84),IY(84),IZ(84)
      C
0009         DATA IX/1,1,1,2,1,1,1,2,2,3,1,1,1,1,2,2,2,3,3,4,1,1,1,1,1,2,2,2,2,
            +3,3,3,4,4,5,1,1,1,1,1,1,2,2,2,2,2,3,3,3,3,4,4,4,5,5,6,1,1,1,1,
            +1,1,1,2,2,2,2,2,2,3,3,3,3,3,4,4,4,4,5,5,5,6,6,7/
0010         DATA IY/1,1,2,1,1,2,3,1,2,1,1,2,3,4,1,2,3,1,2,1,1,2,3,4,5,1,2,3,4,
            +1,2,3,1,2,1,1,2,3,4,5,6,1,2,3,4,5,1,2,3,4,1,2,3,1,2,1,1,2,3,4,
            +5,6,7,1,2,3,4,5,6,1,2,3,4,5,1,2,3,4,1,2,3,1,2,1/
0011         DATA IZ/1,2,1,1,3,2,1,2,1,1,4,3,2,1,3,2,1,2,1,1,5,4,3,2,1,4,3,2,1,
            +3,2,1,2,1,1,6,5,4,3,2,1,5,4,3,2,1,4,3,2,1,3,2,1,2,1,1,7,6,5,4,
            +3,2,1,6,5,4,3,2,1,5,4,3,2,1,4,3,2,1,3,2,1,2,1,1/
      C
0012         DEFINE FILE 1(251,2016,U,IPOINT)
      C
      C    COMPARE COORDINATES TO REGION BOUNDARIES.  IF POINT IS OUTSIDE FINITE
      C    ELEMENT FIELD, PRINT ERROR MESSAGE AND CALL FIMASS
      C
0013         IF((H.LT.HMIN).OR.(H.GT.HMAX).OR.(ALAM.LT.ALMIN).OR.(ALAM.GT.ALMAX
            +).OR.(APHI.LT.APMIN).OR.(APHI.GT.APMAX)) GO TO 400
0014         ISITIN=1
      C
      C    COMPUTE VARIOUS INDEXES TO HELP FIND THE RIGHT CELL IN THE REGION
      C
0015         I=IDINT((H-HMIN)/HCELL+1.D0)
0016         J=IDINT((ALAM-ALMIN)/ALCELL+1.D0)
0017         K=IDINT((APHI-APMIN)/APCELL+1.D0)
0018         IF(H.EQ.HMAX) I=I-1
0019         IF(ALAM.EQ.ALMAX) J=J-1
0020         IF(APHI.EQ.APMAX) K=K-1
0021         KI=I
0022         KJ=J
0023         KK=K
      C
      C    CALCULATE THE RECORD POINTER FOR THE SET OF COEFFICIENTS AT THESE
      C    COORDINATES
```

```
                 C
0024                   IPOINT=(I-1)*NLAM*NPHI+(J-1)*NPHI+K+1
0025                   IF (IPOINT .EQ. IFLAG)  GOTO 200
                 C
                 C   READ A NEW SET OF COEFFICIENTS ONLY IF PROPER SET IS NOT IN CORE
                 C
0026                   IFLAG=IPOINT
0027                   READ(1'IPOINT) (CU(IC),CE(IC),CN(IC),IC=1,NC)
                 C
                 C   FIND THE MINIMUM CELL BOUNDARY COORDINATES
                 C
0028               200 HMCELL=HMIN+(RI-1.D0)*HCELL
0029                   ALMCEL=ALMIN+(RJ-1.D0)*ALCELL
0030                   APMCEL=APMIN+(RK-1.D0)*APCELL
                 C
                 C   NORMALIZE THE CALLING COORDINATES
                 C
0031                   X1=(H-HMCELL)/HCELL
0032                   X2=(ALAM-ALMCEL)/ALCELL
0033                   X3=(APHI-APMCEL)/APCELL
                 C
                 C   EVALUATE THE BASIS FUNCTIONS AT THE TEST POINT FOR EACH ORDER
                 C
0034                   CALL CHEBY(X1,NORDER,TX)
0035                   CALL CHEBY(X2,NORDER,TY)
0036                   CALL CHEBY(X3,NORDER,TZ)
                 C
                 C   EVALUATE THE 3 COORDINATE GRAVITY DISTURBANCES
                 C
0037                   GU=0.D0
0038                   GE=0.D0
0039                   GN=0.D0
                 C
0040                   DO 300  II=1,NC
0041                   NX=IX(II)
0042                   NY=IY(II)
0043                   NZ=IZ(II)
                 C
                 C   DETERMINE THE CONTRIBUTION THAT EACH COEFFICIENT HAS UPON THE FINAL
                 C   DISTURBANCE BY MULTIPLYING THE COEFFICIENTS BY THE APPROPRIATE
                 C   BASIS FUNCTIONS
                 C
0044                   AAA=TX(NX)*TY(NY)*TZ(NZ)
0045                   GU=GU+AAA*CU(II)
0046                   GE=GE+AAA*CE(II)
0047                   GN=GN+AAA*CN(II)
0048               300 CONTINUE
0049                   RETURN
0050               400 CONTINUE
                 C
                 C   IF THE TEST POINT DOES NOT LIE WITHIN THE REGION MODELED IN SECTION
                 C   II THE GRAVITY DISTURBANCE MUST BE EVALUATED BY MODEL SIG
                 C
0051                   CALL PTMASS
0052                   COSL=DCOS(ALAM)
```

```
      SINL=DSIN(ALAM)
      COSP=DCOS(APHI)
      SINP=DSIN(APHI)
      GU=COSP*COSL*GX + COSP*SINL*GY + SINP*GZ
      GE=-SINL*GX + COSL*GY
      GN=-SINP*COSL*GX - SINP*SINL*GY + COSP*GZ
      WRITE(6,1)
    1 FORMAT(49H POINT NOT IN FINITE ELEMENT FIELD, PTMASS CALLED)
      ISITIN=0
      RETURN
      END
```

```
0001              SUBROUTINE CHEBY(X,N,TA)
            C-----------------------------------------------------------------------
            C
            C
            C     BY              JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
            C            DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
            C            SCHOOL OF ENGINEERING
            C            VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
            C            BLACKSBURG, VIRGINIA    24061
            C
            C     DATE OF LAST MODIFICATION -- APRIL 1,1979
            C
            C-----------------------------------------------------------------------
            C
            C
            C-----------------------------------------------------------------------
            C
            C  INPUTS
            C
            C  FROM SUBROUTINE CALL
            C  X--THE NORMALIZED COORDINATE WHERE THE FUNCTION IS TO BE EVALUATED
            C  N--ORDER OF THE BASIS FUNCTION DESIRED
            C
            C
            C  PROCESS
            C
            C  THIS SUBROUTINE EVALUATES THE BASIS FUNCTION AT THE NORMALIZED
            C  COORDINATE VALUE X, FOR EACH ORDER OF THE FUNCTION SPECIFIED (N).
            C  THE FUNCTION VALUES ARE RETURNED IN VECTOR TA.
            C
            C  OUTPUTS
            C
            C  TO SUBROUTINE RETURN
            C  TA--FUNCTION VALUES OF ORDER N EVALUATED AT X
            C
            C-----------------------------------------------------------------------
0002              IMPLICIT REAL*8 (A-H,O-Z)
0003              DIMENSION  TA(7)
            C
            C  S/R CHEBY RETURNS CHEBYCHEV POLYNOMIALS THROUGH ORDER N,
            C  EVALUATED AT X, IN VECTOR TA
            C
0004              IF (N .GT. 0 .AND. N .LT. 7) GOTO 10
0005              WRITE(6,5) N
0006            5 FORMAT(24H ILLEGAL NORDER, NORDER=,I6)
0007              STOP
0008           10 CONTINUE
            C
            C  TRANSFORM THE COORDINATE RANGE (0,1) TO (-1,1)
            C
0009              XBAR=2.D0*X-1.D0
0010              TA(1)=1.D0
0011              TA(2)=XBAR
0012              NP1=N+1
0013              DO 20 I=3,NP1
```

```
0014          20 TA(I)=2.*XBAR*TA(I-1)-TA(I-2)
0015             RETURN
0016             END
```

```
FORTRAN IV G1   RELEASE 2.0               PTMASS               DATE = 79206          20/34/03
 0001                 SUBROUTINE PTMASS
                C-----------------------------------------------------------------------
                C
                C
                C     BY                      JOHN L. JUNKINS AND JOHN SAUNDERS
                C
                C     MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
                C           DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
                C           SCHOOL OF ENGINEERING
                C           VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
                C           BLACKSBURG, VIRGINIA    24061
                C
                C     DATE OF LAST MODIFICATION -- APRIL 1,1979
                C
                C-----------------------------------------------------------------------
                C
                C  FOR P,  SOME POINT IN SPACE
                C  DISTURBANCE IS TO BE EVALUATED
                C  XI,YI,ZI--EARTH-FIXED RECTANGULAR COORDINATES OF THE ITH POINT MASS
                C  POSITS(I,1)=XI
                C  POSITS(I,2)=YI
                C  POSITS(I,3)=ZI
                C
                C  INPUTS
                C
                C  FROM COMMON XYZ
                C  X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES OF POINT AT WHICH GRAVITY
                C  FROM COMMON MASPOS
                C  PMVALS--PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL CONSTANT AND THE
                C     1080 POINT MASSES (-1.E19, 0., OR +1.E19)
                C  POSITS--PRECOMPUTED EARTH-FIXED X,Y,Z COORDINATES OF THE 1080 POINT
                C     MASSES
                C
                C  PROCESS
                C
                C  GIVEN THE RECTANGULAR COORDINATES, X, Y, Z, OF A POINT, THIS ROUTINE
                C  RETURNS THE COMPONENTS OF THE GRAVITY DISTURBANCE, DELGX, DELGY, AND
                C  DELGZ, USING THE POINT MASSES OF MASS MODEL 310.
                C
                C  OUTPUTS
                C
                C  TO COMMON XYZ
                C  DELGX,DELGY,DELGZ--EARTH-FIXED RECTANGULAR COMPONENTS OF THE GRAVITY
                C     DISTURBANCE
                C
                C-----------------------------------------------------------------------
 0002                 IMPLICIT REAL*8 (A-H,O-Z)
 0003                 COMMON  /XYZ/X,Y,Z,DELGX,DELGY,DELGZ
 0004                 COMMON  /MASPOS/PMVALS(1080),POSITS(1080,3)
                C
 0005                 DELGX=0.D0
 0006                 DELGY=0.D0
 0007                 DELGZ=0.D0
                C
 0008                 DO 20 I=1,1080
```

```
FORTRAN IV G1  RELEASE 2.0              PIMASS              DATE = 79206          20/34/03          PA
 0009                  IF (PMVALS(I)) 10,20,10
 0010               10 CONTINUE
 0011                  DX=POSITS(I,1)-X
 0012                  DY=POSITS(I,2)-Y
 0013                  DZ=POSITS(I,3)-Z
 0014                  DISTSQ=DX*DX+DY*DY+DZ*DZ
 0015                  DIST=DSQRT(DISTSQ)
 0016                  TEMP=PMVALS(I)/DIST/DISTSQ
            C
 0017                  DELGX=DELGX+(DX*TEMP)
 0018                  DELGY=DELGY+(DY*TEMP)
 0019                  DELGZ=DELGZ+(DZ*TEMP)
            C
 0020               20 CONTINUE
            C
 0021                  RETURN
 0022                  END
```

1.	75.10	-29.90
270001.	75.90	-29.10

H (KM)	LAMBDA (DEG)	PHI (DEG)	ERRGU (MGALS)	ERRGE (MGALS)	ERRGN (MGALS)	PMGU (MGALS)	PMGE (MGALS)	PMGN (MGALS)	FIGU (MGALS)	FIGE (MGALS)	FIGN (MGALS)
0.001	75.1000	-29.9000	0.3926	0.6070	0.0101	-0.2254	13.4039	7.2174	-0.6180	12.7969	7.2073
0.001	75.1000	-29.6333	0.1204	1.3752	0.2248	-2.9006	14.3385	4.8139	-3.0210	12.9633	4.5890
0.001	75.1000	-29.3667	0.1494	0.6506	-0.0160	-3.4434	13.0140	1.0287	-3.5929	12.3633	1.0446
0.001	75.1000	-29.1000	0.2308	-1.1554	0.1823	-2.3244	9.8959	-0.8016	-2.5552	11.0513	-0.9839
0.001	75.3667	-29.9000	-0.6630	0.2810	0.8330	-4.9833	13.1130	8.5330	-4.3203	12.8320	7.7000
0.001	75.3667	-29.6333	-1.8626	0.7334	0.3964	-8.6290	14.0756	4.9061	-6.7664	13.3422	4.5097
0.001	75.3667	-29.3667	-0.8244	0.4408	-1.1649	-8.0765	13.6860	-0.0188	-7.2520	13.2452	1.1461
0.001	75.3667	-29.1000	1.6151	-0.3063	-1.1007	-4.3775	12.2898	-3.0293	-5.9926	12.5961	-1.9286
0.001	75.6333	-29.9000	-0.3242	-0.6524	1.1615	-7.8117	11.6532	9.6725	-7.4875	12.3056	8.5110
0.001	75.6333	-29.6333	-1.9111	-0.4254	0.5449	-12.1227	12.6666	5.0980	-10.2116	13.0920	4.5530
0.001	75.6333	-29.3667	-0.9667	0.0254	-1.2696	-11.8045	13.4652	-0.0574	-10.8378	13.4398	1.2122
0.001	75.6333	-29.1000	1.3861	0.8457	-1.0967	-8.2015	14.2375	-3.2884	-9.5876	13.3918	-2.1917
0.001	75.9000	-29.9000	0.7690	-0.5360	1.0738	-9.3644	10.7489	10.6460	-10.1334	11.2850	9.5723
0.001	75.9000	-29.6333	-1.1101	-0.6452	0.6248	-14.4745	11.6489	6.4125	-13.3644	12.2941	5.7877
0.001	75.9000	-29.3667	-0.8127	-0.2476	-0.7673	-15.1766	12.7691	1.0020	-14.3639	13.0167	1.7694
0.001	75.9000	-29.1000	0.0548	0.8059	-0.2601	-13.2986	14.3131	-2.1007	-13.3534	13.5072	-1.8405
90.001	75.1000	-29.9000	-0.2144	-0.6369	-0.2866	-0.8264	4.8236	2.2272	-0.6120	5.4605	2.5140
90.001	75.1000	-29.6333	0.4726	-0.6606	-0.0746	-1.5009	4.9204	1.6352	-1.9737	5.5811	1.7096
90.001	75.1000	-29.3667	0.2506	-0.2466	0.3260	-1.9641	4.8658	0.8746	-2.2147	5.1126	0.5486
90.001	75.1000	-29.1000	-0.6288	0.5457	0.3685	-2.1851	4.6552	0.0410	-1.5563	4.1095	-0.3275
90.001	75.3667	-29.9000	0.3644	-0.2892	-0.4174	-2.0568	5.2666	2.3625	-2.4212	5.5558	2.7799
90.001	75.3667	-29.6333	0.9872	-0.5093	0.0730	-2.7841	5.3380	1.6655	-3.7713	5.8472	1.5925
90.001	75.3667	-29.3667	0.6242	-0.4238	0.6985	-3.2429	5.2859	0.7733	-3.8671	5.7097	0.0748
90.001	75.3667	-29.1000	-0.4754	-0.0919	0.9467	-3.4055	5.1056	-0.1762	-2.9301	5.1974	-1.1229
90.001	75.6333	-29.9000	0.4414	0.1779	-0.7212	-3.3298	5.4765	2.4615	-3.7712	5.2985	3.1827
90.001	75.6333	-29.6333	1.2265	-0.1872	-0.0477	-4.1140	5.5116	1.0816	-5.3405	5.6989	1.1293
90.001	75.6333	-29.3667	0.9271	-0.3669	0.7126	-4.5949	5.4632	0.6931	-5.5220	5.8301	-0.0196
90.001	75.6333	-29.1000	-0.2160	-0.4270	1.0693	-4.7433	5.3197	-0.3524	-4.5273	5.7466	-1.4217
90.001	75.9000	-29.9000	0.0967	0.7177	-1.1536	-4.5792	5.4759	2.5009	-4.6758	4.7582	3.6545
90.001	75.9000	-29.6333	1.2870	0.2380	-0.3851	-5.4081	5.4433	1.6733	-6.6951	5.2053	2.0584
90.001	75.9000	-29.3667	1.2643	-0.1720	0.4263	-5.9289	5.3713	0.6240	-7.1932	5.5433	0.1977
90.001	75.9000	-29.1000	0.2897	-0.5865	0.7885	-6.1019	5.2401	-0.4972	-6.3916	5.8265	-1.2856
180.001	75.1000	-29.9000	0.2171	-0.1652	0.3293	-1.0209	2.0133	0.8436	-1.2380	2.1785	0.5143
180.001	75.1000	-29.6333	0.4411	-0.2999	-0.0402	-1.2589	2.0389	0.5379	-1.6999	2.3388	0.5781
180.001	75.1000	-29.3667	0.3179	-0.0820	-0.0782	-1.4335	2.0054	0.1757	-1.7514	2.0874	0.2539
180.001	75.1000	-29.1000	0.0842	0.4347	-0.4147	-1.5296	1.9134	-0.2506	-1.6138	1.4787	0.1641
180.001	75.3667	-29.9000	0.3927	-0.0146	0.2023	-1.5856	2.2220	0.8885	-1.9785	2.2366	0.6862
180.001	75.3667	-29.6333	0.5437	-0.2221	0.0455	-1.8282	2.2337	0.5351	-2.3720	2.4558	0.4896
180.001	75.3667	-29.3667	0.2251	-0.1701	0.1880	-1.9964	2.1931	0.1238	-2.2214	2.3632	-0.0642
180.001	75.3667	-29.1000	-0.3252	0.0277	0.0030	-2.0736	2.1009	-0.3301	-1.7485	2.0733	-0.3330
180.001	75.6333	-29.9000	0.1512	0.0741	0.1043	-2.1845	2.3468	0.9180	-2.3357	2.2727	0.8136
180.001	75.6333	-29.6333	0.4049	-0.0264	0.0446	-2.4267	2.3422	0.5224	-2.8315	2.3686	0.4779
180.001	75.6333	-29.3667	0.1838	-0.0802	0.2527	-2.5862	2.2927	0.0683	-2.7699	2.3728	-0.1844
180.001	75.6333	-29.1000	-0.4527	-0.1407	0.1637	-2.6450	2.1989	-0.4271	-2.1924	2.3396	-0.5908
180.001	75.9000	-29.9000	-0.4722	0.2021	0.1011	-2.7956	2.3790	0.9247	-2.3234	2.1769	0.8236
180.001	75.9000	-29.6333	0.2411	0.2091	0.0229	-3.0314	2.3560	0.4979	-3.2725	2.1469	0.4750
180.001	75.9000	-29.3667	0.2317	0.1091	0.1819	-3.1789	2.2955	0.0076	-3.4106	2.1864	-0.1743
180.001	75.9000	-29.1000	-0.2594	-0.1488	-0.0459	-3.2187	2.1984	-0.5236	-2.9593	2.3472	-0.4777
270.001	75.1000	-29.9000	0.2685	0.0189	-0.0472	-0.9443	0.7014	0.3334	-1.2128	0.6825	0.3806
270.001	75.1000	-29.6333	-0.1180	-0.2445	-0.2191	-1.0345	0.7235	0.1472	-0.9164	0.9680	0.3663
270.001	75.1000	-29.3667	-0.1720	-0.3039	0.0056	-1.0918	0.7153	-0.0613	-0.9198	1.0192	-0.0669
270.001	75.1000	-29.1000	0.3346	-0.2116	-0.0158	-1.1099	0.6789	-0.2876	-1.4445	0.8905	-0.2718
270.001	75.3667	-29.9000	0.4957	0.0186	-0.2254	-1.2133	0.8055	0.3660	-1.7090	0.7868	0.5914
270.001	75.3667	-29.6333	-0.0152	-0.0755	-0.2878	-1.3025	0.8237	0.1536	-1.2873	0.8992	0.4415
270.001	75.3667	-29.3667	-0.2215	-0.1233	0.0168	-1.3535	0.8137	-0.0798	-1.1320	0.9371	-0.0966
270.001	75.3667	-29.1000	-0.1954	-0.1777	0.2517	-1.3598	0.7774	-0.3283	-1.1644	0.9551	-0.5800
270.001	75.6333	-29.9000	0.3992	-0.0878	-0.1820	-1.4986	0.8720	0.3941	-1.8979	0.9597	0.5762
270.001	75.6333	-29.6333	0.0166	0.0468	-0.2545	-1.5651	0.8856	0.1574	-1.5817	0.8388	0.4119
270.001	75.6333	-29.3667	-0.3301	0.0737	0.0107	-1.6286	0.8732	-0.0990	-1.2984	0.7995	-0.1097
270.001	75.6333	-29.1000	-0.3533	-0.0650	-0.0225	-1.6228	0.8371	-0.3690	-1.2695	0.9021	-0.3465
270.001	75.9000	-29.9000	0.0021	-0.3735	0.1502	-1.7910	0.8972	0.4173	-1.7931	1.2707	0.2671
270.001	75.9000	-29.6333	-0.0593	0.0555	-0.0517	-1.8727	0.9059	0.1561	-1.8134	0.8504	0.2078
270.001	75.9000	-29.3667	-0.1145	0.2153	0.0549	-1.9075	0.8910	-0.1191	-1.7930	0.6757	-0.1740
270.001	75.9000	-29.1000	-0.1157	0.0541	-0.1655	-1.8892	0.8549	-0.4076	-1.7735	0.8009	-0.2421

	ERRGU	ERRGE	ERRGN
MEANS	0.0720	-0.0415	0.0728
STANDARD DEVIATIONS	0.6646	0.4316	0.5249
ABS. VALUES OF MAX. ERRORS	1.9111	1.3752	1.2696

FINITE TIME/EXECUTION= 0.0

PIMASS TIME/EXECUTION= 0.045957

ORTHONORMAL POLYNOMIAL FINITE ELEMENT SOFTWARE

```
C      PROGRAM LOCALG(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE2,TAPE3) *
C-----------------------------------------------------------------------
C
C    SECTION IIA
C      LOCAL GRAVITY MODEL -- COEFFICIENT DETERMINATION PROGRAM
C
C      BY                    JOHN L. JUNKINS AND JOHN SAUNDERS
C
C      MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C            DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C            SCHOOL OF ENGINEERING
C            VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C            BLACKSBURG, VIRGINIA    24061
C
C      DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C-----------------------------------------------------------------------
C   FOR P,   SOME POINT IN SPACE
C   X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES
C   H,ALAM,APHI--ELLIPSOIDAL COORDINATES
C   RN--EAST-WEST RADIUS OF CURVATURE
C   NORDER--ORDER OF THE BASIS FUNCTIONS DESIRED
C   H--HEIGHT ABOVE REF. ELLIPSOID ALONG NORMAL TO P
C   ALAM--ANGLE LAMBDA, GEODETIC/GEOCENTRIC LONGITUDE (RADIANS) OF P
C   APHI--ANGLE PHI, GEODETIC LATITUDE (RADIANS) OF P
C
C   INPUTS
C
C   FROM CARDS
C  FORMAT 1
C   NORDER--ORDER OF POLYNOMIALS DESIRED
C   MOBSU,MOBSE,MOBSN--OBSERVATION GRID PATTERN FOR ONE CELL
C
C   THE CARD INPUT FOR THIS SAMPLE RUN IS AS FOLLOWS:
C          3           4           4           4
C
C   FORMAT 2
C   HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
C   HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
C
C   THE CARD INPUT FOR THIS SAMPLE RUN IS AS FOLLOWS:
C300000.                  1.                     1.
C 0.                      75.                    -30.
C 300000.                 76.                    -29.
C
C   FROM DISK
C   FILE 2
C   PMVALS--PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL CONSTANT AND THE
C      1080 POINT MASSES (-1.E19, 0., OR +1.E19)
C   POSITS--PRECOMPUTED EARTH-FIXED X,Y,Z COORDINATES OF THE 1080 POINT
C      MASSES
C
C   PROCESS
C
```

```
C   THIS PROGRAM ACCEPTS CARD INPUT WHICH DIMENSIONS THE REGION TO BE
C   MODELED, THE FINITE ELEMENT GRID PATTERN TO BE USED, AND THE ORDER
C   OF THE BASIS FUNCTIONS THAT ARE TO BE USED IN THE MODELING.  BY
C   ESTABLISHING A STANDARD FINITE ELEMENT SIZE THE ENTIRE AREA IS
C   BROKEN DOWN INTO A NUMBER OF CELLS AND EACH CELL IS MODELED
C   SEPARATELY BY THE SAME PROCEDURE.  THE INPUT SPECIFICATIONS AND
C   FINITE ELEMENT CELL DATA ARE THEN WRITTEN INTO THE FIRST RECORD OF
C   A SEQUENTIAL ACCESS FILE (FILE 3).  BY USING A STANDARD FINITE
C   ELEMENT CELL THE BASIS FUNCTIONS ARE EVALUATED AT EACH GRID POINT
C   FOR EACH ORDER OF THE POLYNOMIALS UP TO THE SPECIFIED ORDER.  SINCE
C   EACH NORMALIZED CELL APPEARS THE SAME, THESE POLYNOMIALS NEED ONLY BE
C   CALCULATED ONCE NO MATTER HOW MANY CELLS MAKE UP THE ENTIRE AREA.
C   VALUES OF THE BASIS FUNCTIONS OF PREDETERMINED ORDERS ARE THEN
C   MULTIPLIED TOGETHER TO FORM A MATRIX WHICH WILL BE USED AS A LEAST
C   SQUARES MATRIX IN SECTION II(B).  THIS MATRIX IS WRITTEN INTO THE
C   SEQUENTIAL ACCESS FILE (FILE 3).   NEXT THE ACTUAL CORDINATES OF
C   EACH GRID POINT OF THE CELL ARE DETERMINED AND THE THREE GRAVITY
C   DISTURBANCE COMPONENTS AT THE POINT ARE FOUND.  IN THIS SOFTWARE THE
C   DISTURBANCES ARE GIVEN BY SUBROUTINE PTMASS WHICH USES MASS MODEL 310
C   TO CALCULATE THE 'OBSERVATIONS'.  THE THREE RECTANGULAR (X,Y,Z) COM-
C   PONENTS ARE THEN CONVERTED TO THEIR ELLIPSOIDAL (H,LAMDA,PHI) VALUES
C   AND THESE ARE WRITTEN ONTO FILE 3.  THIS PROCESS IS REPEATED FOR EACH
C   CELL UNTIL ALL COMPONENTS OF ALL THE OBSERVATION POINTS HAVE BEEN
C   RECORDED.
C
C   OUTPUT
C
C   TO DISK
C   FILE 3
C
C   HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
C   HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
C   P1,P2,P3--THE COEFFICIENTS OF THE ORTHONORMAL BASIS FUNCTIONS
C   IN EACH DIRECTION
C   NH,NLAM,NPHI--NUMBER OF CELLS IN H,LAMDA,PHI DIRECTION
C   NORDER--ORDER OF POLYNOMIALS DESIRED
C   NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
C   MOBS--THE TOTAL NUMBER OF OBSERVATIONS MADE IN EACH CELL
C   NCELLS--THE TOTAL NUMBER OF CELLS IN THE MODELING AREA
C   A--THE LEAST SQUARE MATRIX
C   DELGU,DELGE,DELGN--THE GRAVITY DISTURBANCE OBSERVATIONS IN EACH
C   COORDINATE DIRECTION
C
C-----------------------------------------------------------------------
          IMPLICIT REAL*8 (A-H,O-Z)
          COMMON  /XYZ/X,Y,Z,GX,GY,GZ
          COMMON  /MASPOS/PMVALS(1080),POSITS(1080,3)
C
          DIMENSION IX(7,7),IY(7,7),IZ(7,7),A(343,84)
          DIMENSION DELGU(343),DELGE(343),DELGN(343)
          DIMENSION  IX(84),IY(84),IZ(84)
          DIMENSION P1(7,2),P2(7,2),P3(7,2)
          DIMENSION ICLL(6)
C
```

```
0009          DATA RA/6378160.D0/
0010          DATA RB/6356774.504D0/
0011          DATA ICOL/4,10,20,35,56,84/
0012          DATA IX/1,1,1,2,1,1,1,2,2,3,1,1,1,1,2,2,2,3,3,4,1,1,1,1,1,2,2,2,2,
             +3,3,3,4,4,5,1,1,1,1,1,1,2,2,2,2,2,3,3,3,3,4,4,4,5,5,6,1,1,1,1,
             +1,1,1,2,2,2,2,2,2,3,3,3,3,3,4,4,4,4,5,5,5,6,6,7/
0013          DATA IY/1,1,2,1,1,2,3,1,2,1,1,2,3,4,1,2,3,1,2,1,1,2,3,4,5,1,2,3,4,
             +1,2,3,1,2,1,1,2,3,4,5,6,1,2,3,4,5,1,2,3,4,1,2,3,1,2,1,1,2,3,4,
             +5,6,7,1,2,3,4,5,6,1,2,3,4,5,1,2,3,4,1,2,3,1,2,1/
0014          DATA IZ/1,2,1,1,3,2,1,2,1,1,4,3,2,1,3,2,1,2,1,1,5,4,3,2,1,4,3,2,1,
             +3,2,1,2,1,1,6,5,4,3,2,1,5,4,3,2,1,4,3,2,1,3,2,1,2,1,1,7,6,5,4,
             +3,2,1,6,5,4,3,2,1,5,4,3,2,1,4,3,2,1,3,2,1,2,1,1/
        C
0015        1 FORMAT(8I10)
0016        2 FORMAT(3E20.14)
0017        3 FORMAT(1H1)
0018        4 FORMAT(1X,12I10)
0019        5 FORMAT(1X,3E20.12)
        C
        C   SET CONSTANTS
        C
0020          PI=DARCOS(-1.D0)
0021          DEGRAD=PI/180.D0
0022          A2=RA*RA
0023          B2=RB*RB
        C
        C   READ THE MASS VALUES AND THEIR X,Y,Z COORDINATES FROM FILE 2
        C
0024          READ(2)  PMVALS,POSITS
        C
        C   READ THE POLYNOMIALS ORDER,GRID PATTERN,CELL SIZE,AND FINITE
        C   ELEMENT FIELD BOUNDS
        C
0025          READ(5,1,END=500) NORDER,MOBSU,MOBSE,MOBSN
0026          READ(5,2,END=500) HCELL,ALCELL,APCELL,HMIN,ALMIN,APMIN,HMAX,
             +ALMAX,APMAX
        C
        C   CALCULATE THE TOTAL NUMBER OF GRAVITY OBSERVATIONS TO BE MADE AND
        C   AND THE CONSTANTS NEEDED LATER IN THE PROGRAM
        C
0027          MOBS=MOBSU*MOBSE*MOBSN
0028          RMU=MOBSU
0029          RME=MOBSE
0030          RMN=MOBSN
0031          RMUM1=RMU-1.D0
0032          RMEM1=RME-1.D0
0033          RMNM1=RMN-1.D0
        C
        C   COMPUTE THE NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
        C
0034          NC=ICOL(NORDER)
        C
        C   OUTPUT THE INPUT AND OBSERVATION GRID DATA
        C
0035          WRITE(6,3)
```

```
0036          WRITE(6,4) NORDER,NC
0037          WRITE(6,4) MOBSU,MOBSE,MOBSN,MOBS
0038          WRITE(6,5) HCELL,ALCELL,APCELL,HMIN,ALMIN,APMIN,HMAX,ALMAX,APMAX
        C
        C   CONVERT ANGLES FROM DEGREES TO RADIANS
        C
0039          ALCELL=ALCELL*DEGRAD
0040          APCELL=APCELL*DEGRAD
0041          ALMIN=ALMIN*DEGRAD
0042          APMIN=APMIN*DEGRAD
0043          ALMAX=ALMAX*DEGRAD
0044          APMAX=APMAX*DEGRAD
        C
        C   COMPUTE THE NUMBER OF FINAL CELLS IN UP,EASTERN, AND NORTHERN
        C   DIRECTION AND THE TOTAL NUMBER OF FINAL CELLS IN THE FINITE
        C   ELEMENT FIELD
        C
0045          RNH=(HMAX-HMIN)/HCELL+.999D0
0046          RNL=(ALMAX-ALMIN)/ALCELL+.999D0
0047          RNP=(APMAX-APMIN)/APCELL+.999D0
0048          NH=RNH
0049          NLAM=RNL
0050          NPHI=RNP
0051          NCELLS=NH*NLAM*NPHI
        C
        C   COMPUTE THE INCREMENT SIZE WITHIN EACH CELL.
        C
0052      110 DH=HCELL/RMUM1
0053          DLAM=ALCELL/RMEM1
0054          DPHI=APCELL/RMNM1
        C
0055          DIH=2.D0/RMUM1
0056          DIL=2.D0/RMEM1
0057          DIP=2.D0/RMNM1
        C
        C   GET THE COEFFICIENTS OF THE ORTHOGONIAL POLYNOMIALS FOR A TYPICAL
        C   CELL
        C
0058          CALL ORTHO(P1,NORDER,MOBSU,DIH)
0059          X1=-1.D0
0060          DO 120 IH=1,MOBSU
0061          CALL MULT(X1,NORDER,P1,TX(1,IH))
0062          X1=X1+DIH
0063      120 CONTINUE
0064          CALL ORTHO(P2,NORDER,MOBSE,DIL)
0065          X2=-1.D0
0066          DO 130 IL=1,MOBSE
0067          CALL MULT(X2,NORDER,P2,TY(1,IL))
0068          X2=X2+DIL
0069      130 CONTINUE
0070          CALL ORTHO(P3,NORDER,MOBSN,DIP)
0071          X3=-1.D0
0072          DO 140 IP=1,MOBSN
0073          CALL MULT(X3,NORDER,P3,TZ(1,IP))
0074          X3=X3+DIP
```

```
FORTRAN IV G1   RELEASE 2.0          MAIN                  DATE = 79206          22/16/52                P
0075                140 CONTINUE
                C
                C-------------------------------------------------------------------------
                C
                C    NOTE:  IF A SYMMETRIC GRID PATTERN IS ALWAYS CHOSEN (SUCH AS 4*4*4)
                C    THE THREE SETS OF EVALUATIONS ABOVE ARE REDUNDANT AND TWO COULD BE
                C    ELIMINATED
                C
                C-------------------------------------------------------------------------
                C
                C    WRITE FIELD DATA ONTO FIRST RECORD OF SEQUENTIAL FILE
                C
0076                    WRITE(3) HMIN,HMAX,HCELL,ALMIN,ALMAX,ALCELL,APMIN,APMAX,APCELL,P1,
                      +P2,P3,NH,NLAM,NPHI,NORDER,NC,MOBS,NCELLS
                C
                C    FILL THE A MATRIX
                C
0077                    DO 150  II=1,NC
0078                    NX=IX(II)
0079                    NY=IY(II)
0080                    NZ=IZ(II)
0081                    MJ=0
0082                    DO 150 IH=1,MOBSO
0083                    DO 150 IL=1,MOBSE
0084                    DO 150 IP=1,MOBSN
0085                    MJ=MJ+1
0086                    A(MJ,II)=TX(NX,IH)*TY(NY,IL)*TZ(NZ,IP)
0087                150 CONTINUE
0088                    WRITE(3) ((A(I,J),J=1,NC),I=1,MOBS)
0089                    HMCELL=HMIN-HCELL
0090                    DO 300 IHC=1,NH
                C
                C    INCREMENT THE H COORDINATE (HMCELL) 1 CELL SIZE
                C
0091                    HMCELL=HMCELL+HCELL
                C
                C    RESET THE LAMDA COORDINATE TO ITS CELL MINIMUM VALUE
                C
0092                    ALMCEL=ALMIN-ALCELL
0093                    DO 300 ILC=1,NLAM
                C
                C    INCREMENT THE LAMDA COORDINATE (ALMCEL) 1 CELL SIZE
                C
0094                    ALMCEL=ALMCEL+ALCELL
                C
                C    RESET THE PHI COORDINATE TO ITS CELL MINIMUM VALUE
                C
0095                    APMCEL=APMIN-APCELL
0096                    DO 300 IPC=1,NPHI
                C
                C    INCREMENT THE PHI COORDINATE (APMCEL) 1 CELL SIZE
                C
0097                    APMCEL=APMCEL+APCELL
0098                    MJ=0
0099                    H=HMCELL-DH
```

```
0100                 DO 250  IH=1,MOBSU
               C
               C   INCREMENT THE H COORDINATE (HMCELL) 1 GRID INCREMENT SIZE
               C
0101                 H=H+DH
0102                 ALAM=ALMCEL-DLAM
0103                 DO 250  IL=1,MOBSE
               C
               C   INCREMENT THE LAMDA COORDINATE (ALMCEL) 1 GRID INCREMENT SIZE
               C
0104                 ALAM=ALAM+DLAM
0105                 COSL=DCOS(ALAM)
0106                 SINL=DSIN(ALAM)
0107                 APHI=APMCEL-DPHI
0108                 DO 250  IP=1,MOBSN
               C
               C   INCREMENT THE PHI COORDINATE (APMCEL) 1 GRID INCREMENT SIZE
               C
0109                 APHI=APHI+DPHI
0110                 COSP=DCOS(APHI)
0111                 SINP=DSIN(APHI)
0112                 MJ=MJ+1
               C
               C   TRANSFORM ALL ELLIPSOIDAL COORDINATES INTO X,Y,Z COORDINATES
               C
0113                 RN=DSQRT(A2*COSP*COSP+B2*SINP*SINP)
0114                 ZN=B2/RN
0115                 RN=A2/RN
0116                 Y=(RN+H)*COSP
0117                 X=Y*COSL
0118                 Y=Y*SINL
0119                 Z=(ZN+H)*SINP
               C
               C   OBTAIN THE GRAVITY 'OBSERVATION' AT THIS GRID POINT
               C
0120                 CALL PTMASS
               C
               C   DIRECTION COSINE MATRIX FOR 3-2 ROTATION (LAMBDA,-PHI) FROM X,Y,Z, TO
               C   UP,EAST,NORTH
               C
               C   C11=COSP*COSL
               C   C12=COSP*SINL
               C   C13=SINP
               C   C21=-SINL
               C   C22=COSL
               C   C23=0
               C   C31=-SINP*COSL
               C   C32=-SINP*SINL
               C   C33=COSP
               C
0121                 GU=COSP*COSL*GX + COSP*SINL*GY + SINP*GZ
0122                 GE=-SINL*GX + COSL*GY
0123                 GN=-SINP*COSL*GX - SINP*SINL*GY + COSP*GZ
               C
               C   STORE THE OBSERVATION DISTURBANCES
```

```
      C
        200 DELGU(MJ)=GU
            DELGE(MJ)=GE
            DELGN(MJ)=GN
        250 CONTINUE
      C
      C
      C   WRITE THE THREE OBSERVATION ARRAYS ONTO THE SEQUENTIAL FILE
      C
            WRITE(3) (DELGU(I),I=1,MOBS)
            WRITE(3) (DELGE(I),I=1,MOBS)
            WRITE(3) (DELGN(I),I=1,MOBS)
        300 CONTINUE
        500 STOP
            END
```

```
FORTRAN IV G1   RELEASE 2.0          ORTHO               DATE = 79206          22/16/52        P
 0001                 SUBROUTINE ORTHO(P,NORDER,MOB,DELTAL)
                C-----------------------------------------------------------------------
                C
                C      BY                   JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
                C
                C            DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
                C            SCHOOL OF ENGINEERING
                C            VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
                C            BLACKSBURG, VIRGINIA    24061
                C
                C      DATE OF LAST MODIFICATION -- APRIL 1,1979
                C
                C-----------------------------------------------------------------------
                C
                C   INPUT
                C
                C   FROM SUBROUTINE CALL
                C   NORDER--ORDER OF BASIS FUNCTIONS DESIRED
                C   MOB--NUMBER OF INTERVALS IN THE FINITE ELEMENT CELL
                C   DELTAL--SIZE OF EACH INTERVAL
                C
                C   PROCESS
                C
                C   THIS SUBROUTINE ACCEPTS THE BASIS FUNCTION ORDER, THE INTERVAL SIZE
                C   AND THE NUMBER OF INTERVALS OF THE FINITE ELEMENT CELL TO BE MODELED.
                C   AFTER THE MATRICES ARE INITIALIZED, THE NORMALIZED INNER PRODUCTS AND
                C   FINAL COEFFICIENTS ARE CALCULATED.  SINCE THE FIRST EQUATION, ORDER 1
                C   IS F1=1.0 AND ORDER 2, F2=X, ARE KNOWN, ONLY THE INNER PRODUCTS ARE
                C   SAVED SO THAT THE RECURSIVE NATURE OF THE POLYNOMIALS CAN BE USED TO
                C   CALCULATE THE BASIS FUNCTION VALUES IN SUBROUTINE MULT.
                C
                C   OUTPUT
                C
                C   TO SUBROUTINE RETURN
                C   P--THE INNER PRODUCTS USED TO CALCULATE THE BASIS FUNCTIONS VALUES
                C      RECURSIVELY
                C
                C-----------------------------------------------------------------------
 0002                 IMPLICIT REAL*8 (A-H,O-Z)
 0003                 DIMENSION F(7,7),P(7,2),BLP(13)
 0004                 N=NORDER+1
 0005                 MOBS=N+NORDER
                C   INITIALIZE THE MATRICES THAT WILL HOLD THE FINAL COEFFICIENTS OF
                C   THE BASIS FUNCTIONS AND THE INNER PRODUCTS
 0006                 DO 20 I=1,N
 0007                 P(I,1)=0.D0
 0008                 P(I,2)=0.D0
 0009                 DO 20 J=1,N
 0010                 F(J,I)=0.D0
 0011              20 CONTINUE
                C   USING THE 'EVEN' PROPERTY OF THE ORTHOGONAL EQUATIONS, CALCULATE THE
                C   SUMS:   BLP=X1**I+X2**I+...XN**I,    WHERE I=2,3,4,...MOBS,   AND
                C      WHERE X=THE NORMALIZED COORDINATE OF EACH GRID POINT
                C   WHICH ARE PART OF THE INNER PRODUCTS OF ONE ORTHOGONAL POLYNOMIAL
                C   TIMES ITSELF.
```

```
0012          BLP(1)=MOB
0013          DO 100 I=2,MOBS
0014          J=I-1
0015          BLP(I)=0.D0
0016          XL=-1.D0-DELTAL
0017          DO 100 K=1,MOB
0018          XL=XL+DELTAL
0019          IF(DABS(XL).LE.1.D-15) GO TO 100
0020          BLP(I)=BLP(I)+XL**J
0021      100 CONTINUE
        C  CALCULATE THE COEFFICIENTS OF THE ORTHOGONAL EQUATIONS OF ORDER I BY
        C  USING THE RECURSIVE PROCESS:  F(I+1)=F(I)*X-P(I)/P(I-1)*F(I-1)
        C  WHERE P(I) IS THE INNER PRODUCT OF F(I)*F(I)
0022          DO 400 I=1,N
0023          F(I,1)=1.D0
        C  CALCULATE THE INNER PRODUCT P(I)
0024          DO 200 K=1,I
0025          DO 200 L=1,I
0026          LP=K+L-1
0027          P(I,1)=P(I,1)+F(K,I)*F(L,I)*BLP(LP)
0028      200 CONTINUE
0029          IF((I.LE.1).OR.(I.GE.N)) GO TO 400
0030          J=I-1
0031          K=I+1
        C  USING THE INNER PRODUCTS CALCULATE THE COEFFICIENTS OF F(I+1).
0032          P(J,2)=P(I,1)/P(J,1)
0033          F(1,K)=-P(J,2)*F(1,J)
0034          DO 300 L=2,J
0035          M=L-1
0036          F(L,K)=F(M,I)-P(J,2)*F(L,J)
0037      300 CONTINUE
0038      400 CONTINUE
        C  NORMALIZE THE COEFFICIENTS OF THE ORTHOGONAL POLYNOMIALS F.
0039          DO 600 J=1,N
0040          P(J,1)=DSQRT(P(J,1))
0041          DO 600 I=1,N
0042          F(I,J)=F(I,J)/P(J,1)
0043      600 CONTINUE
0044          RETURN
0045          END
```

```
FORTRAN IV G1   RELEASE 2.0            MULT                 DATE = 79206          22/16/52        P
      SUBROUTINE MULT(X,N,P,TA)
C-----------------------------------------------------------------------------
C
C     BY                    JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C
C           DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C           SCHOOL OF ENGINEERING
C           VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C           BLACKSBURG, VIRGINIA   24061
C
C     DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C-----------------------------------------------------------------------------
C
C  INPUTS
C
C  FROM SUBROUTINE CALL
C  X--THE NORMALIZED COORDINATE WHERE THE FUNCTION IS TO BE EVALUATED
C  N--ORDER OF THE BASIS FUNCTION DESIRED
C  P--THE INNER PRODUCTS USED TO CALCULATE THE BASIS FUNCTIONS VALUES
C     RECURSIVELY
C
C  PROCESS
C
C  THIS SUBROUTINE EVALUATES THE BASIS FUNCTIONS AT THE NORMALIZED CO-
C  ORDINATE VALUE X, FOR EACH ORDER OF THE BASIS FUNCTIONS SPECIFIED(N).
C  THE FUNCTION VALUES ARE RETURNED IN VECTOR TA.  THE VALUES ARE
C  CALCULATED BY USING THE RECURSIVE RELATIONSHIP:
C  TA(I+1)=(TA(I)*X-P(I)/P(I-1)*TA(I-1))/P(I)**.5
C
C  OUTPUTS
C
C  TO SUBROUTINE RETURN
C  TA--FUNCTION VALUES OF ORDER N EVALUATED AT X
C
C-----------------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION P(7,2),TA(7)
      NP1=N+1
      TA(1)=1.D0
      TA(2)=X
      DO 100 I=3,NP1
      J=I-1
      K=I-2
      TA(I)=(X*TA(J)-P(K,2)*TA(K))
      TA(K)=TA(K)/P(K,1)
  100 CONTINUE
      TA(N)=TA(N)/P(N,1)
      TA(NP1)=TA(NP1)/P(NP1,1)
      RETURN
      END
```

```
0001              SUBROUTINE PTMASS
            C-----------------------------------------------------------------------
            C
            C
            C     BY                JOHN L. JUNKINS AND JOHN SAUNDERS
            C
            C     MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
            C        DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
            C        SCHOOL OF ENGINEERING
            C        VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
            C        BLACKSBURG, VIRGINIA    24061
            C
            C     DATE OF LAST MODIFICATION -- APRIL 1,1979
            C
            C-----------------------------------------------------------------------
            C
            C FOR P,  SOME POINT IN SPACE
            C DISTURBANCE IS TO BE EVALUATED
            C X1,Y1,Z1--EARTH-FIXED RECTANGULAR COORDINATES OF THE ITH POINT MASS
            C POSITS(I,1)=X1
            C POSITS(I,2)=Y1
            C POSITS(I,3)=Z1
            C
            C INPUTS
            C
            C FROM COMMON XYZ
            C X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES OF POINT AT WHICH GRAVITY
            C FROM COMMON MASPOS
            C PMVALS--PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL CONSTANT AND THE
            C    1080 POINT MASSES (-1.E19, 0., OR +1.E19)
            C POSITS--PRECOMPUTED EARTH-FIXED X,Y,Z COORDINATES OF THE 1080 POINT
            C    MASSES
            C
            C PROCESS
            C
            C GIVEN THE RECTANGULAR COORDINATES, X, Y, Z, OF A POINT, THIS ROUTINE
            C RETURNS THE COMPONENTS OF THE GRAVITY DISTURBANCE, DELGX, DELGY, AND
            C DELGZ, USING THE POINT MASSES OF MASS MODEL 310.
            C
            C OUTPUTS
            C
            C TO COMMON XYZ
            C DELGX,DELGY,DELGZ--EARTH-FIXED RECTANGULAR COMPONENTS OF THE GRAVITY
            C    DISTURBANCE
            C
            C-----------------------------------------------------------------------
0002              IMPLICIT REAL*8 (A-H,O-Z)
0003              COMMON /XYZ/X,Y,Z,DELGX,DELGY,DELGZ
0004              COMMON  /MASPOS/PMVALS(1080),POSITS(1080,3)
            C
0005              DELGX=0.D0
0006              DELGY=0.D0
0007              DELGZ=0.D0
            C
0008              DO 20 I=1,1080
```

```
      IF (PMVALS(I)) 10,20,10
   10 CONTINUE
      DX=POSITS(I,1)-X
      DY=POSITS(I,2)-Y
      DZ=POSITS(I,3)-Z
      DISTSQ=DX*DX+DY*DY+DZ*DZ
      DIST=DSQRT(DISTSQ)
      TEMP=PMVALS(I)/DIST/DISTSQ
C
      DELGX=DELGX+(DX*TEMP)
      DELGY=DELGY+(DY*TEMP)
      DELGZ=DELGZ+(DZ*TEMP)
C
   20 CONTINUE
C
      RETURN
      END
```

```
       3              20
       4               4              4          64
0.300000000000D+06   0.100000000000D+01   0.100000000000D+01
0.0                  0.750000000000D+02  -0.300000000000D+02
0.300000000000D+06   0.760000000000D+02  -0.290000000000D+02
```

```
C-----------------------------------------------------------------------
C
C     BY                JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C
C          DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C          SCHOOL OF ENGINEERING
C          VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C          BLACKSBURG, VIRGINIA   24061
C
C     DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C-----------------------------------------------------------------------
C
C   INPUTS
C
C   FROM DISK
C   FILE 3
C   HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
C   HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
C   P1,P2,P3--THE COEFFICIENTS OF THE ORTHONORMAL BASIS FUNCTIONS
C   IN EACH DIRECTION
C   NH,NLAM,NPHI--NUMBER OF CELLS IN H,LAMDA,PHI DIRECTION
C   NORDER--ORDER OF POLYNOMIALS DESIRED
C   NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
C   MOBS--THE TOTAL NUMBER OF OBSERVATIONS MADE IN EACH CELL
C   NCELLS--THE TOTAL NUMBER OF CELLS IN THE MODELING AREA
C   A--THE LEAST SQUARE MATRIX
C   DELGU,DELGE,DELGN--THE GRAVITY DISTURBANCE OBSERVATIONS IN EACH
C   COORDINATE DIRECTION
C
C   PROCESS
C
C   THIS PROGRAM ACCEPTS AS INPUT DATA WHICH DESCRIBES THE REGION TO BE
C   MODELED AND THE COEFFICIENTS OF THE ORTHONORMAL BASIS FUNCTIONS, FROM
C   DISK FILE 3.  THIS INFORMATION IS IN TURN WRITTEN AS THE FIRST RECORD
C   OF THE RANDOM ACCESS FILE (FILE 1).  THE LEAST SQUARES MATRIX A, IS
C   ALSO READ FROM FILE 3.  NEXT EACH GRAVITY DISTURBANCE VECTOR IS READ
C   FROM FILE 3 AND IT, ALONG WITH THE LEAST SQUARES MATRIX IS PASSED TO
C   SUBROUTINE ALSQ1.  SUBROUTINE ALSQ1 MULTIPLIES THE TRANSPOSE OF THE
C   LEAST SQUARES MATRIX BY THE DISTURBANCE VECTOR TO PRODUCT THE MODEL-
C   ING COEFFICIENTS WHICH 'CORRECTLY' FIT THE BASIS FUNCTIONS TO THE
C   OBSERVATIONS.  THE THREE SETS OF COEFFICIENTS OF EACH CELL ARE THEN
C   WRITTEN INTO THE NEXT AVAILABLE RECORD OF FILE 1.  IF MORE THAN ONE
C   CELL IS INVOLVED THE COEFFICIENTS ARE WRITTEN IN THE NORMAL ORDER
C   OF PHI VARIES MOST RAPIDLY, FOLLOWED BY LAMDA AND FINALLY H.
C
C   OUTPUT
C
C   TO DISK
C   FILE 1
C
C   HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
C   HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
```

```
            C  P1,P2,P3--THE COEFFICIENTS OF THE ORTHONORMAL BASIS FUNCTIONS
            C  IN EACH DIRECTION
            C  NH,NLAM,NPHI--NUMBER OF CELLS IN H,LAMDA,PHI DIRECTION
            C  NORDER--ORDER OF POLYNOMIALS DESIRED
            C  NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
            C  CU,CE,CN--THE MODELING COEFFICIENTS
            C
            C-----------------------------------------------------------------------
0001              IMPLICIT REAL*8 (A-H,O-Z)
0002              DIMENSION P1(7,2),P2(7,2),P3(7,2)
0003              DIMENSION A(344,85),DELG(343)
0004              DIMENSION CU(84),CE(84),CN(84)
0005              DATA  MAXOB1/344/
0006              DEFINE FILE 1(251,2016,U,IPOINT)
            C
0007            6 FORMAT(1X,3E12.4)
0008            7 FORMAT(1H0,10X,18HALSQ CALLED, TIME=,F10.4)
0009            8 FORMAT(1H0,10X,16HCOEFFICIENTS FOR,I5,25H CELLS COMPUTED, DELTAT=
                 +,F10.4,14H AVERAGE TIME=,F10.4)
0010              CALL TIMEON
            C
0011              READ(3) HMIN,HMAX,HCELL,ALMIN,ALMAX,ALCELL,APMIN,APMAX,APCELL,P1,
                 +P2,P3,NH,NLAM,NPHI,NORDER,NC,MOBS,NCELLS
0012              WRITE(1'1) HMIN,HMAX,HCELL,ALMIN,ALMAX,ALCELL,APMIN,APMAX,APCELL,
                 +P1,P2,P3,NH,NLAM,NPHI,NORDER,NC
            C
0013              KNCELL=NCELLS
            C
            C  READ THE LEAST SQUARES MATRIX FROM THE SEQUENTIAL FILE
            C
0014              READ(3) ((A(I,J),J=1,NC),I=1,MOBS)
            C
            C  REDUCE THE LEAST SQUARES MATRIX TO UPPER TRIANGULAR FORM
            C
0015              CALL TIMECK(ICP)
0016              TIME=ICP/100.D0
0017              WRITE(6,7) TIME
            C
            C  FOR EACH CELL, READ ONE SET OF GRAVITY OBSERVATIONS AND
            C  COMPUTE THE COEFFICIENT ARRAY FOR EACH COMPONENT OF GRAVITY
            C
0018              CALL TIMEON
0019              DO 40 ID=1,NCELLS
0020              READ(3) (DELG(I),I=1,MOBS)
0021              CALL ALSQ1(A,DELG,CU,MOBS,NC,MAXOB1)
            C
0022              READ(3) (DELG(I),I=1,MOBS)
0023              CALL ALSQ1(A,DELG,CE,MOBS,NC,MAXOB1)
            C
0024              READ(3) (DELG(I),I=1,MOBS)
0025              CALL ALSQ1(A,DELG,CN,MOBS,NC,MAXOB1)
            C
            C  STORE EACH COEFFICIENT ARRAY IN A RANDOM ACCESS FILE
            C
0026              WRITE(1'IPOINT) (CU(IC),CE(IC),CN(IC),IC=1,NC)
```

```
   40 CONTINUE
      CALL TIMECK(ICP)
      TIME=ICP/100.D0
      TAV=TIME/RNCELL
      WRITE(6,8) NCELLS,TIME,TAV
   50 STOP
      END
```

```
      SUBROUTINE ALSQ1(A,DELG,C,MOBS,NC,MAXOB1)
C-----------------------------------------------------------------------
C
C     BY              JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C
C            DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C            SCHOOL OF ENGINEERING
C            VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C            BLACKSBURG, VIRGINIA   24061
C
C     DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C-----------------------------------------------------------------------
C
C  INPUTS
C
C  FROM SUBROUTINE CALL
C
C  A--THE LEAST SQUARE MATRIX
C  DELGU,DELGE,DELGN--THE GRAVITY DISTURBANCE OBSERVATIONS IN EACH
C  COORDINATE DIRECTION
C  MOBS--THE TOTAL NUMBER OF OBSERVATIONS MADE IN EACH CELL
C  NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
C  MAXOB1--THE MAXIMUM NUMBER OF OBSERVATIONS IN EACH CELL
C
C  PROCESS
C
C  THIS SUBROUTINE ACCEPTS FROM THE MAIN PROGRAM LOCALG THE LEAST SQUARE
C  MATRIX A, A VECTOR OF GRAVITY DISTURBANCES AND THE DIMENSIONS OF THE
C  TWO ARRAYS.  THE TRANSPOSE OF THE LEAST SQUARES MATRIX IS THEN
C  MULTIPLIED BY THE DISTURBANCE VECTOR AND THE RESULTING VECTOR IS THE
C  SET OF MODELING COEFFICEINTS THAT BEST FIT THE BASIS FUNCTIONS THAT
C  MAKE UP THE LEAST SQUARES MATRIX TO THE DISTURBANCE OBSERVATIONS.
C  THESE COEFFICIENTS ARE THEN RETURNED TO THE CALLING ROUTINE.
C
C  OUTPUT
C
C  C--THE MODELING COEFFICIENTS
C
C-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(MAXOB1,1),DELG(1),C(1)
      N1=MOBS+1
      DO 100 J=1,NC
      A(J,N1)=0.D0
      DO 50 I=1,MOBS
      A(J,N1)=A(J,N1)+A(I,J)*A(I,J)
      C(J)=C(J)+A(I,J)*DELG(I)
   50 CONTINUE
      C(J)=C(J)/A(J,N1)
  100 CONTINUE
      RETURN
      END
```

```
ALSQ CALLED, TIME=     0.1900
COEFFICIENTS FOR    1 CELLS COMPUTED, DELTAT=      0.0700 AVERAGE TIME=     0.0700
```

```
C     PROGRAM FINTES(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1,TAPE2) *
C-----------------------------------------------------------------------
C
C   SECTION III
C     LOCAL GRAVITY MODEL -- ERROR ANALYSIS  AND TIME COMPARISON
C
C     BY                 JOHN L. JUNKINS AND JOHN SAUNDERS
C
C     MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C           DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C           SCHOOL OF ENGINEERING
C           VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C           BLACKSBURG, VIRGINIA   24061
C
C     DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C-----------------------------------------------------------------------
C
C    FOR P,  SOME POINT IN SPACE
C    X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES
C    H,ALAM,APHI--ELLIPSOIDAL COORDINATES
C    RN--EAST-WEST RADIUS OF CURVATURE
C    NORDER--ORDER OF THE BASIS FUNCTIONS DESIRED
C    H--HEIGHT ABOVE REF. ELLIPSOID ALONG NORMAL TO P
C    ALAM--ANGLE LAMBDA, GEODETIC/GEOCENTRIC LONGITUDE (RADIANS) OF P
C    APHI--ANGLE PHI, GEODETIC LATITUDE (RADIANS) OF P
C
C    INPUTS
C
C    FROM DISK
C    FILE 1
C    HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
C    HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
C    HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
C    P1,P2,P3--THE COEFFICIENTS OF THE ORTHONORMAL BASIS FUNCTIONS
C    IN EACH DIRECTION
C    NH,NLAM,NPHI--NUMBER OF CELLS IN H,LAMDA,PHI DIRECTION
C    NORDER--ORDER OF POLYNOMIALS DESIRED
C    NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
C
C    FILE 2
C    PMVALS--PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL CONSTANT AND THE
C       1080 POINT MASSES (-1.E19, 0., OR +1.E19)
C    POSITS--PRECOMPUTED EARTH-FIXED X,Y,Z COORDINATES OF THE 1080 POINT
C       MASSES
C
C    FROM CARDS
C    FORMAT 1
C    ISTEPH,ISTEPL,ISTEPP--TEST GRID PATTERN
C
C    THE CARD INPUT FOR THIS SAMPLE RUN IS AS FOLLOWS:
C     4     4     4
C
C    FORMAT 2
C    HMIN,ALMIN,APMIN--MINUMUM BOUNDS OF TEST FIELD DESIRED
```

```
FORTRAN IV G1   RELEASE 2.0          MAIN                DATE = 79206          22/17/43

C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF TEST FIELD DESIRED
C
C   THE CARD INPUT FOR THIS SAMPLE RUN IS AS FOLLOWS:
C 1.         75.1         -29.9
C2/0(C1.     75.9         -29.1
C
C   PROCESS
C
C   THIS PROGRAM ACCEPTS CARD INPUT WHICH SPECIFIES THE MINIMUN AND
C   MAXIMUM LIMITS OF A TEST REGION WHICH OVERLAPS OR IS CONTAINED IN THE
C   REGION ESTABLISHED AND MODELLED IN SECTION II.  THE ENTIRE REGION
C   CONSTITUTES A SINGLE CELL WHICH IS AGAIN DIVIDED BY A REGULAR GRID
C   PATTERN.  AT EACH GRID OR OBSERVATION POINT THE ACTUAL GRAVITY
C   DISTURBANCES ARE CALCULATED USING MASS MODEL 310 (SUBROUTINE PIMASS)
C   AND THE MODELLED GRAVITY DISTURBANCES ARE CALCULATED BY USING THE
C   MODELLING EQUATION WHOSES COEFFICIENTS WERE DETERMINED IN SECTION
C   II AND STORED ON A RANDOM ACCESS FILE (FILE 1).AN ERROR ANALYSIS IS
C   PERFORMED ON EACH TEST POINT AND THE RESULTS ARE OUTPUT.  IN ADDI-
C   TION TO THE POINT BY POINT ANALYSIS, THE MEANS, STANDARD DIVIATIONS,
C   AND MAXIMUM ABSOLUTE ERRORS ARE OUTPUT ALONG WITH THE TOTAL
C   EXECUTION TIMES OF THE TWO METHODS.
C
C   OUTPUTS
C
C   ERROR ANALYSIS REPORT
C
C-------------------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON  /HLP/H,ALAM,APHI,GU,GE,GN
      COMMON  /XYZ/X,Y,Z,GX,GY,GZ
      COMMON/IMARK/IFLAG,ISITIN
      COMMON/FLDATA/ REA(9),REB(14),REC(14),RED(14),INT(5)
      COMMON  /MASPOS/PMVALS(1080),POSITS(1080,3)
C
      DATA A/6378160.D0/
      DATA B/6356774.504D0/
C
      DEFINE FILE 1(251,2016,U,IPOINT)
C
    1 FORMAT(16I5)
    2 FORMAT(F10.0,2F10.2)
    3 FORMAT(1H1)
    4 FORMAT(1X,12I10)
    5 FORMAT(1X,F10.0,2F10.2)
    6 FORMAT(1H0,4X,1HH,8X,6HLAMBDA,5X,3HPHI,7X,5HERRGU,5X,5HERRGE,5X
     +5HERRGN,5X,4HPMGU,6X,4HPMGE,6X,4HPMGN,6X,4HFIGU,6X,4HFIGE,6X,
     +4HFIGN)
    7 FORMAT(4X,4H(KM),1X,2(5X,5H(DEG)),2X,9(3X,7H(MGALS)))
    8 FORMAT(2X,9(1H-),11(2X,8(1H-)))
    9 FORMAT(1H0,10X,22HFINITE TIME/EXECUTION=,F9.6)
   10 FORMAT(1H0,10X,22HPIMASS TIME/EXECUTION=,F9.6)
   11 FORMAT(1X,-3PF10.3,2(0PF10.4),9(1X,5PF9.4))
   12 FORMAT(///,1X,5HMEANS,25X,3(1X,5PF9.4))
   13 FORMAT(1X,19HSTANDARD DEVIATIONS,11X,3(1X,5PF9.4))
   14 FORMAT(1X,26HABS. VALUES OF MAX. ERRORS,4X,3(1X,5PF9.4))
```

```
0024          15 FORMAT(46H0*****      NEGATIVE OR ZERO NO. OF STEPS     *****)          *
           C
           C     READ IN THE FIELD DATA FROM THE FIRST RECORD OF THE RANDOM ACCESS
           C     FILE (FILE 1).
           C
0025             READ(1'1) KEA,KEB,KEC,KED,INT
           C
           C     READ FILE 2 WHICH CONTAINS THE MASS VALUES AND THEIR X,Y,Z COORDINATE
           C     SET UP CONVERSION FACTORS AND INITIALIZATION
           C
0026             READ(2) PMVALS,POSITS
0027             PI=DARCOS(-1.D0)
0028             DEGRAD=PI/180.D0
0029             A2=A*A
0030             B2=B*B
0031             IFLAG=0
0032             ISITIN=0
           C
           C     READ IN THE TEST CELL GRID PATTERN AND THE FINITE ELEMENT FIELD
           C     BOUNDS
           C
0033          20 READ(5,1,END=40) ISTEPH,ISTEPL,ISTEPP
0034             READ(5,2,END=40) HMIN,ALMIN,APMIN,HMAX,ALMAX,APMAX
0035             WRITE(6,3)
0036             WRITE(6,4) ISTEPH,ISTEPL,ISTEPP
0037             WRITE(6,5) HMIN,ALMIN,APMIN,HMAX,ALMAX,APMAX
0038             IHM1=ISTEPH-1
0039             ILM1=ISTEPL-1
0040             IPM1=ISTEPP-1
           C
           C     VERIFY THAT THE TEST CELL GRID PATTERN IS VALID AND CALCULATE THE
           C     GRID INCREMENT SIZE
           C
0041             IF (IHM1)   21,22,23
0042          21 WRITE(6,15)
0043             GOTO 40
0044          22 DH=0.D0
0045             GOTO 24
0046          23 RHM1=IHM1
0047             DH=(HMAX-HMIN)/RHM1
0048          24 IF (ILM1)   21,25,26
0049          25 DLAM=0.D0
0050             GOTO 27
0051          26 RLM1=ILM1
0052             DLAM=(ALMAX-ALMIN)/RLM1
0053          27 IF (IPM1)   21,28,29
0054          28 DPHI=0.D0
0055             GOTO 30
0056          29 RPM1=IPM1
0057             DPHI=(APMAX-APMIN)/RPM1
           C
           C     SET UP THE ERROR ANALYSIS REPORT HEADINGS
           C
0058          30 WRITE(6,6)
0059             WRITE(6,7)
```

```
0060         WRITE(6,8)
       C
       C   INITIALIZE ALL TIME AND ERROR ANALYSIS VARIABLES
       C
0061         FTIME=0.D0
0062         PTIME=0.D0
0063         EGU=0.D0
0064         EGE=0.D0
0065         EGN=0.D0
0066         SDEGU=0.D0
0067         SDEGE=0.D0
0068         SDEGN=0.D0
0069         EGUMAX=0.D0
0070         EGEMAX=0.D0
0071         EGNMAX=0.D0
       C
       C   DETERMINE THE 3 GRAVITY DISTURBANCES AT EACH TEST GRID POINT
       C
0072         H=HMIN-DH
0073         DO 35  I=1,ISTEPH
       C
       C   INCREMENT THE H COORDINATE 1 GRID INCREMENT SIZE
       C
0074         H=H+DH
0075         AL=ALMIN-DLAM
0076         DO 35  J=1,ISTEPL
       C
       C   INCREMENT THE LAMDA COORDINATE 1 GRID INCREMENT SIZE
       C
0077         AL=AL+DLAM
0078         ALAM=AL*DEGRAD
0079         COSL=DCOS(ALAM)
0080         SINL=DSIN(ALAM)
0081         AP=APMIN-DPHI
0082         DO 35  K=1,ISTEPP
       C
       C   INCREMENT THE PHI COORDINATE 1 GRID INCREMENT SIZE
       C
0083         AP=AP+DPHI
0084         APHI=AP*DEGRAD
       C
       C   TRANSFORM THE ELLIPSOIDAL COORDINATES INTO X,Y,Z
       C
0085         COSP=DCOS(APHI)
0086         SINP=DSIN(APHI)
0087         RN=DSQRT(A2*COSP*COSP + B2*SINP*SINP)
0088         ZN=B2/RN
0089         RN=A2/RN
0090         Y=(RN+H)*COSP
0091         X=Y*COSL
0092         Y=Y*SINL
0093         Z=(ZN+H)*SINP
       C
       C   EVALUATE THE GRAVITY DISTURBANCES USING THE MODELING EQUATION
       C   WHOSE COEFFICIENTS WHERE DETERMINED IN SECTION II
```

```
                C
                C
                C     START THE TIMER TO DETERMINE THE EXECUTION TIME OF THE EQUATION
                C
0094                  CALL TIMEON
0095                  CALL FINITE
                C
                C     STOP THE TIMER AND CONVERT THE TIME TO SECONDS
                C
0096                  CALL TIMECK(ICP)
0097                  FTIME=FTIME + ICP/100.D0
                C
0098                  GUF=GU
0099                  GEF=GE
0100                  GNF=GN
                C
                C     START TIMER TO DETERMINE THE EXECUTION TIME
                C
0101                  CALL TIMEON
                C
                C     EVALUATE THE GRAVITY DISTURBANCES USING MASS MODEL 310
                C
0102                  CALL PTMASS
0103                  CALL TIMECK(ICP)
0104                  PTIME=PTIME + ICP/100.D0
                C
                C     TRANSFORM THE GRAVITY DISTURBANCES TO ELLIPSOIDAL COORDINATES
                C
0105                  GU=COSP*COSL*GX + COSP*SINL*GY + SINP*GZ
0106                  GE=-SINL*GX + COSL*GY
0107                  GN=-SINP*COSL*GX - SINP*SINL*GY + COSP*GZ
                C
                C     DETERMINE THE ERROR VALUE BETWEEN THE TWO METHODS
                C
0108                  ERRGU=GU-GUF
0109                  ERRGE=GE-GEF
0110                  ERRGN=GN-GNF
                C
                C     OUTPUT THE ERROR ANALYSIS FOR THIS GRID POINT
                C
0111                  WRITE(6,11) H,AL,AP,ERRGU,ERRGE,ERRGN,GU,GE,GN,GUF,GEF,GNF
                C
                C     DETERMINE THE MAXIMUM ABSOLUTE ERROR VALUE FOR EACH COORDINATE
                C     DISTURBANCE
                C
0112                  AERRGU=DABS(ERRGU)
0113                  AERRGE=DABS(ERRGE)
0114                  AERRGN=DABS(ERRGN)
0115                  IF(AERRGU.GT.EGUMAX)  EGUMAX=AERRGU
0116                  IF(AERRGE.GT.EGEMAX)  EGEMAX=AERRGE
0117                  IF(AERRGN.GT.EGNMAX)  EGNMAX=AERRGN
                C
                C     CALCULATE THE SUM OF THE ERROR FOR EACH COORDINATE DISTURBANCE
                C
0118                  EGU=EGU+ERRGU
```

```
      EGE=EGE+ERRGE
      EGN=EGN+ERRGN
C
C  CALCULATE THE SUM OF THE SQUARES OF THE ERRORS FOR EACH
C  COORDINATE DISTURBANCE
C
      SDEGU=SDEGU+ERRGU*ERRGU
      SDEGE=SDEGE+ERRGE*ERRGE
      SDEGN=SDEGN+ERRGN*ERRGN
   35 CONTINUE
C
C  CALCULATE THE MEAN AND STANDARD DEVIATION OF THE ERRORS
C
      IPTS=ISTEPH*ISTEPL*ISTEPP
      RIPTS=IPTS
      RIPTM1=RIPTS-1.D0
      EGU=EGU/RIPTS
      EGE=EGE/RIPTS
      EGN=EGN/RIPTS
      SDEGU=DSQRT(SDEGU/RIPTM1)
      SDEGE=DSQRT(SDEGE/RIPTM1)
      SDEGN=DSQRT(SDEGN/RIPTM1)
C
C  OUTPUT THE MEANS, STANDARD DEVIATION AND MAXIMUM ABSOLUTE ERROR
C
      WRITE(6,12) EGU,EGE,EGN
      WRITE(6,13) SDEGU,SDEGE,SDEGN
      WRITE(6,14) EGUMAX,EGEMAX,EGNMAX
      FTIME=FTIME/RIPTS
      PTIME=PTIME/RIPTS
C
C  OUTPUT THE AVERAGE EXECUTION TIME PER POINT FOR EACH METHOD
C
      WRITE(6,9) FTIME
      WRITE(6,10) PTIME
      GOTO 20
   40 CONTINUE
      STOP
      END
```

```
0001          SUBROUTINE FINITE
      C--------------------------------------------------------------------
      C
      C
      C      BY                      JOHN L. JUNKINS AND JOHN SAUNDERS
      C
      C      MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
      C            DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
      C            SCHOOL OF ENGINEERING
      C            VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
      C            BLACKSBURG, VIRGINIA    24061
      C
      C      DATE OF LAST MODIFICATION -- APRIL 1,1979
      C
      C--------------------------------------------------------------------
      C   FOR P,  SOME POINT IN SPACE
      C   X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES
      C   H,ALAM,APHI--ELLIPSOIDAL COORDINATES
      C   RN--EAST-WEST RADIUS OF CURVATURE
      C   NORDER--ORDER OF THE BASIS FUNCTIONS DESIRED
      C
      C   INPUTS
      C
      C   FROM COMMON FLDATA
      C   HMIN,ALMIN,APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED
      C   HMAX,ALMAX,APMAX--MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED
      C   HCELL,ALCELL,APCELL--CELL SIZE IN H,LAMBDA,PHI
      C   NH,NLAM,NPHI--NUMBER OF CELLS IN H,LAMDA,PHI DIRECTION
      C   NORDER--ORDER OF POLYNOMIALS DESIRED
      C   NC--NUMBER OF COEFFICIENTS IN THE MODELING EQUATION
      C   FROM DISK
      C   FILE 1
      C   CU,CE,CN--THE MODELING COEFFICIENTS
      C   FROM COMMON HLP
      C   H--HEIGHT ABOVE REF. ELLIPSOID ALONG NORMAL TO P
      C   ALAM--ANGLE LAMBDA, GEODETIC/GEOCENTRIC LONGITUDE (RADIANS) OF P
      C   APHI--ANGLE PHI, GEODETIC LATITUDE (RADIANS) OF P
      C
      C   PROCESS
      C
      C   THIS PROGRAM ACCEPTS INPUT OF ELLIPSOIDAL COORDINATES (H,LAMDA,PHI)
      C   FROM THE MAIN PROGRAM FINTES AND EVALUATES THE THREE GRAVITY
      C   ANOMALIES AT THIS POINT.  FIRST THE POINT IS TESTED TO SEE IF IT
      C   LIES WITHIN THE REGION MODELED IN SECTION II.  IF NOT, THE GRAVITY
      C   DISTURBANCES ARE EVALUATED BY SUBROUTINE PTMASS WHICH USES MASS
      C   MODEL 310, AND A MESSAGE IS PRINTED INDICATING THIS FACT.  THE CELL
      C   (IF THERE IS MORE THAN ONE CELL THAT MAKES UP THE REGION) WHICH
      C   CONTAINS THIS POINT IS DETERMINED AND THE CORE MEMORY IS CHECKED TO
      C   SEE IF THE MODELING COEFFICIENTS FOR THIS CELL ARE AVAILABLE IN CORE.
      C   IF NOT, THE COEFFICIENTS ARE READ IN FROM FILE 1 WHICH WAS CREATED
      C   IN SECTION II.  WHEN THE COEFFICIENTS ARE AVAILABLE THE COORDINATES
      C   ARE NORMALIZED AND THE BASIS FUNCTIONS AND THEIR FIRST DERIVATIVES
      C   ARE EVALUATED AT THIS POINT FOR EACH ORDER OF THE POLYNOMIALS UP TO
      C   THE SPECIFIED ORDER.  EACH COEFFICIENT IS THEN MULTIPLIED BY THREE
      C   DIFFERENT SETS OF TWO BASIS FUNCTIONS AND ONE DERIVATIVE OF PREDETER-
```

```
              C   MINED ORDERS.  THIS GIVES THE CONTRIBUTION OF EACH COEFFICIENT TO
              C   EACH OF THE THREE GRAVITY ANOMALIES.  WHEN THE THREE CONTRIBUTIONS
              C   OF ALL THE COEFFICIENTS ARE SUMMED SEPARATELY, THE RESULTS ARE THE
              C   X,Y,Z COMPONENTS OF THE GRAVITY ANOMALY AT THE GIVEN POINT.  THESE
              C   COMPONENTS ARE RETURNED TO THE MAIN PROGRAM FINITES.
              C
              C   OUTPUTS
              C
              C   TO COMMON XYZ
              C   GX,GY,GZ--EARTH FIXED RECTANGULAR COMPONENTS OF THE GRAVITY
              C   DISTURBANCE
              C
              C------------------------------------------------------------------------
0002                 IMPLICIT REAL*8 (A-H,O-Z)
0003                 COMMON  /HLP/H,ALAM,APHI,GU,GE,GN
0004                 COMMON  /XYZ/X,Y,Z,GX,GY,GZ
0005                 COMMON/IMARK/IFLAG,ISITIN
0006                 COMMON /FLDATA/HMIN,HMAX,HCELL,ALMIN,ALMAX,ALCELL,APMIN,APMAX,
                    +APCELL,P1(7,2),P2(7,2),P3(7,2),NH,NLAM,NPHI,NORDER,NC
              C
0007                 DIMENSION IX(7),IY(7),IZ(7),CU(84),CE(84),CN(84)
0008                 DIMENSION  IX(84),IY(84),IZ(84)
              C
0009                 DATA IX/1,1,1,2,1,1,1,2,2,3,1,1,1,1,2,2,2,3,3,4,1,1,1,1,1,2,2,2,2,
                    +3,3,3,4,4,5,1,1,1,1,1,1,2,2,2,2,2,3,3,3,3,4,4,4,5,5,6,1,1,1,1,
                    +1,1,1,2,2,2,2,2,2,3,3,3,3,3,4,4,4,4,5,5,5,6,6,7/
0010                 DATA IY/1,1,2,1,1,2,3,1,2,1,1,2,3,4,1,2,3,1,2,1,1,2,3,4,5,1,2,3,4,
                    +1,2,3,1,2,1,1,2,3,4,5,6,1,2,3,4,5,1,2,3,4,1,2,3,1,2,1,1,2,3,4,
                    +5,6,7,1,2,3,4,5,6,1,2,3,4,5,1,2,3,4,1,2,3,1,2,1/
0011                 DATA IZ/1,2,1,1,3,2,1,2,1,1,4,3,2,1,3,2,1,2,1,1,5,4,3,2,1,4,3,2,1,
                    +3,2,1,2,1,1,6,5,4,3,2,1,5,4,3,2,1,4,3,2,1,3,2,1,2,1,1,7,6,5,4,
                    +3,2,1,6,5,4,3,2,1,5,4,3,2,1,4,3,2,1,3,2,1,2,1,1/
              C
0012                 DEFINE FILE 1(251,2016,U,IPOINT)
              C
              C   COMPARE COORDINATES TO REGION BOUNDARIES.  IF POINT IS OUTSIDE FINITE
              C   ELEMENT FIELD, PRINT ERROR MESSAGE AND CALL PTMASS
              C
0013                 IF((H.LT.HMIN).OR.(H.GT.HMAX).OR.(ALAM.LT.ALMIN).OR.(ALAM.GT.ALMAX
                    +).OR.(APHI.LT.APMIN).OR.(APHI.GT.APMAX)) GO TO 400
              C
              C   COMPUTE VARIOUS INDEXES TO HELP FIND THE RIGHT CELL IN THE REGION
              C
0014                 I=IDINT((H-HMIN)/HCELL+1.D0)
0015                 J=IDINT((ALAM-ALMIN)/ALCELL+1.D0)
0016                 K=IDINT((APHI-APMIN)/APCELL+1.D0)
0017                 IF(H.EQ.HMAX) I=I-1
0018                 IF(ALAM.EQ.ALMAX) J=J-1
0019                 IF(APHI.EQ.APMAX) K=K-1
0020                 RI=I
0021                 RJ=J
0022                 RK=K
              C
              C   CALCULATE THE RECORD POINTER FOR THE SET OF COEFFICIENTS AT THESE
              C   COORDINATES
```

```
            C
0023              IPOINT=(I-1)*NLAM*NPHI+(J-1)*NPHI+K+1
0024              IF (IPOINT .EQ. IFLAG)  GOTO 200
            C
            C   READ A NEW SET OF COEFFICIENTS ONLY IF PROPER SET IS NOT IN CORE
            C
0025              IFLAG=IPOINT
0026              READ(1'IPOINT) (CU(IC),CE(IC),CN(IC),IC=1,NC)
            C
            C   FIND THE MINIMUM CELL BOUNDARY COORDINATES
            C
0027          200 HMCELL=HMIN+(RI-1.D0)*HCELL
0028              ALMCEL=ALMIN+(RJ-1.D0)*ALCELL
0029              APMCEL=APMIN+(RK-1.D0)*APCELL
            C
            C   NORMALIZE THE CALLING COORDINATES
            C
0030              X1=(H-HMCELL)/HCELL
0031              X2=(ALAM-ALMCEL)/ALCELL
0032              X3=(APHI-APMCEL)/APCELL
            C
            C   TRANSFORM THE COORDINATE RANGE (0,1) TO (-1,1)
            C
0033              X1=2.D0*X1-1.D0
0034              X2=2.D0*X2-1.D0
0035              X3=2.D0*X3-1.D0
            C
            C   EVALUATE THE BASIS FUNCTIONS AT THE TEST POINT FOR EACH ORDER
            C
0036              CALL MULT(X1,NORDER,P1,TX)
0037              CALL MULT(X2,NORDER,P2,TY)
0038              CALL MULT(X3,NORDER,P3,TZ)
            C
            C   EVALUATE THE 3 COORDINATE GRAVITY DISTURBANCES
            C
0039              GU=0.D0
0040              GE=0.D0
0041              GN=0.D0
            C
0042              DO 300  II=1,NC
0043              NX=IX(II)
0044              NY=IY(II)
0045              NZ=IZ(II)
            C
            C   DETERMINE THE CONTRIBUTION THAT EACH COEFFICIENT HAS UPON THE FINAL
            C   DISTURBANCE BY MULTIPLYING THE COEFFICIENTS BY THE APPROPRIATE
            C   BASIS FUNCTIONS
            C
0046              AAA=TX(NX)*TY(NY)*TZ(NZ)
0047              GU=GU+AAA*CU(II)
0048              GE=GE+AAA*CE(II)
0049              GN=GN+AAA*CN(II)
0050          300 CONTINUE
0051              RETURN
0052          400 CONTINUE
```

```
FORTRAN IV G1   RELEASE 2.0          FINITE          DATE = 79206          22/17/43          PA
      C
      C   IF THE TEST POINT DOES NOT LIE WITHIN THE REGION MODELED IN SECTION
      C   II THE GRAVITY DISTURBANCE MUST BE EVALUATED BY MODEL 310
      C
            CALL PIMASS
            COSL=DCOS(ALAM)
            SINL=DSIN(ALAM)
            COSP=DCOS(APHI)
            SINP=DSIN(APHI)
            GU=COSP*COSL*GX + COSP*SINL*GY + SINP*GZ
            GE=-SINL*GX + COSL*GY
            GN=-SINP*COSL*GX - SINP*SINL*GY + COSP*GZ
            WRITE(6,1)
          1 FORMAT(49H POINT NOT IN FINITE ELEMENT FIELD, PIMASS CALLED)
            ISITIN=0
            RETURN
            END
```

```
      SUBROUTINE MULT(X,N,P,TA)
C-----------------------------------------------------------------------
C
C     BY                      JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
C
C            DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
C            SCHOOL OF ENGINEERING
C            VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
C            BLACKSBURG, VIRGINIA   24061
C
C     DATE OF LAST MODIFICATION -- APRIL 1,1979
C
C-----------------------------------------------------------------------
C
C  INPUTS
C
C  FROM SUBROUTINE CALL
C  X--THE NORMALIZED COORDINATE WHERE THE FUNCTION IS TO BE EVALUATED
C  N--ORDER OF THE BASIS FUNCTION DESIRED
C  P--THE INNER PRODUCTS USED TO CALCULATE THE BASIS FUNCTIONS VALUES
C     RECURSIVELY
C
C  PROCESS
C
C  THIS SUBROUTINE EVALUATES THE BASIS FUNCTIONS AT THE NORMALIZED CO-
C  ORDINATE VALUE X, FOR EACH ORDER OF THE BASIS FUNCTIONS SPECIFIED(N).
C  THE FUNCTION VALUES ARE RETURNED IN VECTOR TA.  THE VALUES ARE
C  CALCULATED BY USING THE RECURSIVE RELATIONSHIP:
C  TA(I+1)=(TA(I)*X-P(I)/P(I-1)*TA(I-1))/P(I)**.5
C
C  OUTPUTS
C
C  TO SUBROUTINE RETURN
C  TA--FUNCTION VALUES OF ORDER N EVALUATED AT X
C
C-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION P(7,2),TA(7)
      NP1=N+1
      TA(1)=1.D0
      TA(2)=X
      DO 100 I=3,NP1
      J=I-1
      K=I-2
      TA(I)=(X*TA(J)-P(K,2)*TA(K))
      TA(K)=TA(K)/P(K,1)
  100 CONTINUE
      TA(N)=TA(N)/P(N,1)
      TA(NP1)=TA(NP1)/P(NP1,1)
      RETURN
      END
```

```
0001          SUBROUTINE PTMASS
        C-----------------------------------------------------------------------
        C
        C
        C     BY                  JOHN L. JUNKINS AND JOHN SAUNDERS
        C
        C     MODIFICATIONS BY JOHN L. JUNKINS,REMI C. ENGELS, AND JOHN J. SMITH
        C          DEPARTMENT OF ENGINEERING SCIENCE AND MECHANICS
        C          SCHOOL OF ENGINEERING
        C          VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
        C          BLACKSBURG, VIRGINIA    24061
        C
        C     DATE OF LAST MODIFICATION -- APRIL 1,1979
        C
        C-----------------------------------------------------------------------
        C
        C  FOR P,  SOME POINT IN SPACE
        C  DISTURBANCE IS TO BE EVALUATED
        C  XI,YI,ZI--EARTH-FIXED RECTANGULAR COORDINATES OF THE ITH POINT MASS
        C  POSITS(I,1)=XI
        C  POSITS(I,2)=YI
        C  POSITS(I,3)=ZI
        C
        C  INPUTS
        C
        C  FROM COMMON XYZ
        C  X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES OF POINT AT WHICH GRAVITY
        C  FROM COMMON MASPOS
        C  PMVALS--PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL CONSTANT AND THE
        C     1080 POINT MASSES (-1.E19, 0., OR +1.E19)
        C  POSITS--PRECOMPUTED EARTH-FIXED X,Y,Z COORDINATES OF THE 1080 POINT
        C     MASSES
        C
        C  PROCESS
        C
        C  GIVEN THE RECTANGULAR COORDINATES, X, Y, Z, OF A POINT, THIS ROUTINE
        C  RETURNS THE COMPONENTS OF THE GRAVITY DISTURBANCE, DELGX, DELGY, AND
        C  DELGZ, USING THE POINT MASSES OF MASS MODEL 310.
        C
        C  OUTPUTS
        C
        C  TO COMMON XYZ
        C  DELGX,DELGY,DELGZ--EARTH-FIXED RECTANGULAR COMPONENTS OF THE GRAVITY
        C     DISTURBANCE
        C
        C-----------------------------------------------------------------------
0002          IMPLICIT REAL*8 (A-H,O-Z)
0003          COMMON  /XYZ/X,Y,Z,DELGX,DELGY,DELGZ
0004          COMMON  /MASPOS/PMVALS(1080),POSITS(1080,3)
        C
0005          DELGA=0.D0
0006          DELGY=0.D0
0007          DELGZ=0.D0
        C
0008          DO 20 I=1,1080
```

```
FORTRAN IV G1   RELEASE 2.0              PIMASS                    DATE = 79206          22/17/43

0009                 IF (PMVALS(I)) 10,20,10
0010              10 CONTINUE
0011                 DX=POSITS(I,1)-X
0012                 DY=POSITS(I,2)-Y
0013                 DZ=POSITS(I,3)-Z
0014                 DISTSQ=DX*DX+DY*DY+DZ*DZ
0015                 DIST=DSQRT(DISTSQ)
0016                 TEMP=PMVALS(I)/DIST/DISTSQ
            C
0017                 DELGX=DELGX+(DX*TEMP)
0018                 DELGY=DELGY+(DY*TEMP)
0019                 DELGZ=DELGZ+(DZ*TEMP)
            C
0020              20 CONTINUE
            C
0021                 RETURN
0022                 END
```

4	4	4
1.	75.10	-29.90
270001.	75.90	-29.10

H (KM)	LAMBDA (DEG)	PHI (DEG)	ERRGU (MGALS)	ERRGE (MGALS)	ERRGN (MGALS)	PMGU (MGALS)	PMGE (MGALS)	PMGN (MGALS)	FIGU (MGALS)	FIGE (MGALS)	FIGN (MGALS)
0.001	75.1000	-29.9000	0.3926	0.6070	0.0101	-6.2254	13.4039	7.2174	-0.6180	12.7969	7.2073
0.001	75.1000	-29.6333	0.1204	1.3752	0.2248	-2.9006	14.3385	4.8139	-3.0210	12.9633	4.5890
0.001	75.1000	-29.3667	0.1494	0.6506	-0.0160	-3.4434	13.0140	1.6287	-3.5929	12.3633	1.6446
0.001	75.1000	-29.1000	0.2308	-1.1554	0.1825	-2.3244	9.6959	-0.8016	-2.5552	11.0513	-0.9633
0.001	75.3667	-29.9000	-0.6631	0.2810	0.8330	-4.5833	13.1132	8.5530	-4.3203	12.8326	7.7000
0.001	75.3667	-29.6333	-1.8600	0.7334	0.3964	-6.6290	14.0756	4.9681	-6.7084	13.3422	4.5717
0.001	75.3667	-29.3667	-0.6244	0.4408	-1.1649	-6.0705	13.6800	-0.0168	-7.2520	13.2452	1.1461
0.001	75.3667	-29.1000	1.6151	-0.3063	-1.1007	-4.3775	12.2898	-3.0293	-5.9926	12.5961	-1.5266
0.001	75.6333	-29.9000	-0.3242	-0.6524	1.1615	-7.6117	11.6532	9.6725	-7.4875	12.3057	8.5110
0.001	75.6333	-29.6333	-1.9111	-0.4294	0.5449	-12.1227	12.6666	5.5360	-10.2116	13.0959	4.5330
0.001	75.6333	-29.3667	-0.9667	0.0254	-1.2696	-11.6045	13.4652	-0.0574	-10.8378	13.4398	1.2122
0.001	75.6333	-29.1000	1.3861	0.8457	-1.0967	-8.2013	14.2373	-3.2684	-9.5874	13.3916	-2.1917
0.001	75.9000	-29.9000	0.7650	-0.5360	1.0738	-9.3644	10.7489	10.8460	-10.1334	11.2850	9.5123
0.001	75.9000	-29.6333	-1.1101	-0.6452	0.6248	-14.4745	11.6489	6.4125	-13.3644	12.2941	5.7877
0.001	75.9000	-29.3667	-0.8129	-0.2476	-0.7673	-15.1768	12.7691	1.0020	-14.3039	13.0167	1.7694
0.001	75.9000	-29.1000	0.0846	0.8059	-0.2601	-13.2686	14.3131	-2.1007	-13.5534	13.5072	-1.2405
90.001	75.1000	-29.9000	-0.2144	-0.6369	-0.2866	-0.8264	4.8236	2.2272	-0.6120	5.4605	2.5140
90.001	75.1000	-29.6333	0.4726	-0.6606	-0.0746	-1.5069	4.9264	1.6352	-1.9737	5.5611	1.7696
90.001	75.1000	-29.3667	0.2506	-0.2406	0.3260	-1.9641	4.8658	0.8746	-2.2147	5.1126	0.5486
90.001	75.1000	-29.1000	-0.6288	0.5457	0.3683	-2.1851	4.0552	0.0416	-1.5563	4.1095	-0.3273
90.001	75.3667	-29.9000	0.3644	-0.2892	-0.4174	-2.0568	5.2666	2.3625	-2.4212	5.5558	2.7755
90.001	75.3667	-29.6333	0.4672	-0.5093	0.0730	-2.7841	5.3380	1.6635	-3.7713	5.6472	1.5905
90.001	75.3667	-29.3667	0.0242	-0.4238	0.6985	-3.2429	5.2859	0.7733	-3.8671	5.7097	0.0748
90.001	75.3667	-29.1000	-0.4154	-0.0919	0.9467	-3.4055	5.1056	-0.1762	-2.9301	5.1974	-1.1249
90.001	75.6333	-29.9000	0.4414	0.1779	-0.7212	-3.3298	5.4765	2.4615	-3.7712	5.2985	3.1827
90.001	75.6333	-29.6333	1.2265	-0.1872	-0.0477	-4.1140	5.5116	1.0816	-5.3405	5.6969	1.7293
90.001	75.6333	-29.3667	0.9271	-0.3669	0.7126	-4.5949	5.4632	0.6931	-5.5220	5.8301	-0.0196
90.001	75.6333	-29.1000	-0.2660	-0.4270	1.0693	-4.7433	5.3197	-0.3524	-4.5573	5.7466	-1.4217
90.001	75.9000	-29.9000	0.0967	0.7177	-1.1536	-4.5792	5.4759	2.5009	-4.6758	4.7582	3.6545
90.001	75.9000	-29.6333	1.2870	0.2380	-0.3851	-5.4081	5.4433	1.6733	-6.6951	5.2333	2.0364
90.001	75.9000	-29.3667	1.2643	-0.1720	0.4263	-5.9289	5.3713	0.6240	-7.1932	5.5433	0.1976
90.001	75.9000	-29.1000	0.2697	-0.5865	0.7685	-6.1019	5.2401	-0.4972	-6.3916	5.8265	-1.2856
180.001	75.1000	-29.9000	0.2171	-0.1652	0.3293	-1.0209	2.0133	0.8436	-1.2380	2.1765	0.5143
180.001	75.1000	-29.6333	0.4411	-0.2999	-0.0402	-1.2589	2.0389	0.5379	-1.6999	2.3388	0.5781
180.001	75.1000	-29.3667	0.3179	-0.0820	-0.0782	-1.4335	2.0054	0.1757	-1.7514	2.6874	0.2539
180.001	75.1000	-29.1000	0.0842	0.4347	-0.4147	-1.5296	1.9134	-0.2366	-1.6138	1.4787	0.1641
180.001	75.3667	-29.9000	0.3927	-0.0146	0.2023	-1.5856	2.2220	0.8885	-1.9785	2.2900	0.0862
180.001	75.3667	-29.6333	0.5437	-0.2221	0.0455	-1.8282	2.2337	0.5351	-2.3720	2.4557	0.4896
180.001	75.3667	-29.3667	0.2251	-0.1761	0.1880	-1.9964	2.1931	0.1258	-2.2214	2.3632	-0.0642
180.001	75.3667	-29.1000	-0.3252	0.0277	0.0030	-2.0736	2.1009	-0.3301	-1.7485	2.0733	-0.3330
180.001	75.6333	-29.9000	0.1512	0.0741	0.1043	-2.1845	2.3466	0.9186	-2.3357	2.2727	0.6136
180.001	75.6333	-29.6333	0.4049	-0.0264	0.0440	-2.4267	2.3422	0.5224	-2.8915	2.3686	0.4779
180.001	75.6333	-29.3667	0.1838	-0.0802	0.2527	-2.5662	2.2927	0.0683	-2.7699	2.3726	-0.1844
180.001	75.6333	-29.1000	-0.4527	-0.1407	0.1037	-2.6450	2.1989	-0.4271	-2.1924	2.3356	-0.5308
180.001	75.9000	-29.9000	-0.4722	0.2027	0.1011	-2.7956	2.3790	0.9297	-2.3234	2.1765	0.1286
180.001	75.9000	-29.6333	0.2411	0.2091	0.0229	-3.0314	2.3560	0.4979	-3.2725	2.1469	0.4750
180.001	75.9000	-29.3667	0.7317	0.1097	0.1219	-3.1789	2.2955	0.0076	-3.4106	2.1656	-0.1143
180.001	75.9000	-29.1000	-0.2594	-0.1488	-0.0459	-3.2187	2.1984	-0.5236	-2.9593	2.3472	-0.4771
270.001	75.1000	-29.9000	0.2685	0.0189	-0.0472	-0.9443	0.7014	0.3334	-1.2128	0.6625	0.3806
270.001	75.1000	-29.6333	-0.1160	-0.2445	-0.2191	-1.0345	0.7235	0.1472	-0.9164	0.9080	0.3063
270.001	75.1000	-29.3667	-0.1720	-0.3039	0.0056	-1.0918	0.7153	-0.0613	-0.9198	1.0192	-0.0669
270.001	75.1000	-29.1000	0.3346	-0.2112	-0.0108	-1.1099	0.6769	-0.2876	-1.4445	0.8905	-0.2766
270.001	75.3667	-29.9000	0.4957	0.0126	-0.2254	-1.2333	0.8655	0.3659	-1.7090	0.7866	0.5914
270.001	75.3667	-29.6333	-0.0152	-0.0755	-0.2878	-1.3025	0.8237	0.1536	-1.2873	0.8992	0.4415
270.001	75.3667	-29.3667	-0.3215	-0.1233	0.0168	-1.3535	0.8137	-0.0796	-1.0320	0.9371	-0.0966
270.001	75.3667	-29.1000	-0.1554	-0.1777	0.2517	-1.3596	0.7774	-0.3289	-1.1644	0.9551	-0.3600
270.001	75.6333	-29.9000	0.3992	-0.0875	-0.1820	-1.4986	0.8720	0.3941	-1.8979	0.9597	0.5762
270.001	75.6333	-29.6333	[illegible]	0.0488	-0.2549	-1.5851	0.8850	0.1574	-1.5817	0.8566	0.4119
270.001	75.6333	-29.3667	-0.3301	0.0737	0.0107	-1.6286	0.8752	-0.0996	-1.2964	0.7795	-0.1097
270.001	75.6333	-29.1000	-0.3533	-0.0650	-0.0225	-1.6228	0.8371	-0.3690	-1.2695	0.9021	-0.3465
270.001	75.9000	-29.9000	0.0021	-0.3735	0.1502	-1.7910	0.6972	0.4173	-1.7931	1.2107	0.2611
270.001	75.9000	-29.6333	-0.0593	0.0555	-0.0517	-1.8727	0.9059	0.1561	-1.6134	0.8564	0.2059
270.001	75.9000	-29.3667	-0.1745	0.2153	0.0549	-1.9075	0.8910	-0.1191	-1.7330	0.6757	-0.1740
270.001	75.9000	-29.1000	-0.1157	0.0541	-0.1655	-1.8892	0.8549	-0.4076	-1.7735	0.8009	-0.2423

	ERRGU	ERRGE	ERRGN
MEANS	0.0720	-0.0415	0.0238
STANDARD DEVIATIONS	0.6646	0.4316	0.5249
ABS. VALUES OF MAX. ERRORS	1.9111	1.3752	1.2696

FINITE TIME/EXECUTION= 0.0

PIMASS TIME/EXECUTION= [illegible]

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ETL-0198	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) FINITE ELEMENT MODELS OF THE EARTH'S GRAVITY FIELD PHASE IV		5. TYPE OF REPORT & PERIOD COVERED Contract Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) R. C. Engels, Assistant Professor J. L. Junkins, Professor, PI		8. CONTRACT OR GRANT NUMBER(s) DAAK-78-C-0072
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Engineering Science & Mechanics Virginia Polytechnic Institute Blacksburg, VA 24061		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Computer Science Laboratory U.S. Army Engineer Topographic Laboratory Fort Belvoir, Virginia 22060		12. REPORT DATE October 5, 1979
		13. NUMBER OF PAGES 150
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Gravity, Finite Element, Guidance, Geodesy

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Methods are developed and demonstrated for constructing piecewise local models for gravity anomalies. The approach is shown to be computationally more efficient (by a factor of about 20) than competing methods. The report includes software listings and test cases.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)